Capture Recapture estimation using finite mixtures of arbitrary dimension

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Outline

- Capture-Recapture experiments and models
- Reversible Jump MCMC
- Application: Software reliability

Capture-Recapture

- k repeated samples taken from a population
- Population size N is unknown
- D distinct individuals are seen
- Some seen on multiple occasions, some only once
- **Goal:** estimate population size N

Capture-Recapture: Applications

- **Size of animal populations**:
  Samples are occasions on which animals are trapped, marked (for re-identification) and released.
  N is the number of animals in the population.

Capture-Recapture: Applications

- **Size of animal populations**:
  Samples are occasions on which animals are trapped, marked (for re-identification) and released.
  N is the number of animals in the population.

- **Software testing**:
  Samples are independent software testers detecting errors.
  N is the number of errors in the piece of software.

Observations: Capture Matrix X

	Samples					
		1	2	3	...	k
Individuals	1	1	0	1	...	0
	2	1	0	1	...	0
	3	0	0	0	...	0
Observed	4	0	1	1	...	0
	5	0	0	1	...	0
	$\vdots$					
	D	0	1	1	...	0
	$D+1$	0	0	0	...	0
	$D+2$	0	0	0	...	0
Unobserved	$\vdots$					
	N	0	0	0	...	0

An unknown number $(N-D)$ of zero rows: to be estimated.

Capture-Recapture

Observations form the $D \times k$ **capture matrix**

$$X_{ij} = \begin{cases} 1 & \text{if individual } i \text{ appears in sample } j \\ 0 & \text{otherwise} \end{cases}$$

Probability of capture

$$\begin{aligned} p_{ij} &= \text{Probability individual } i \text{ is captured in sample } j \\ &= p(X_{ij} = 1) \end{aligned}$$

$$X_{ij} \sim \text{Bernoulli}(p_{ij})$$

$$p(X|P,N) = \frac{N!}{\prod_{\mathbf{x}} N_{\mathbf{x}}!} \prod_{i=1}^{N} \prod_{j=1}^{k} p_{ij}^{x_{ij}} (1-p_{ij})^{1-x_{ij}}$$

Some key assumptions

1. All individuals are catchable
2. **Closed population**: no births/deaths or migration
3. No loss of marks (identifiable individuals)
4. No impact of sampling on capture probabilities
(i.e. no behavioural responses)

(Models do exist for violations of 2-4)

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- **Both sources** – model M_{th}

2. Identifiability

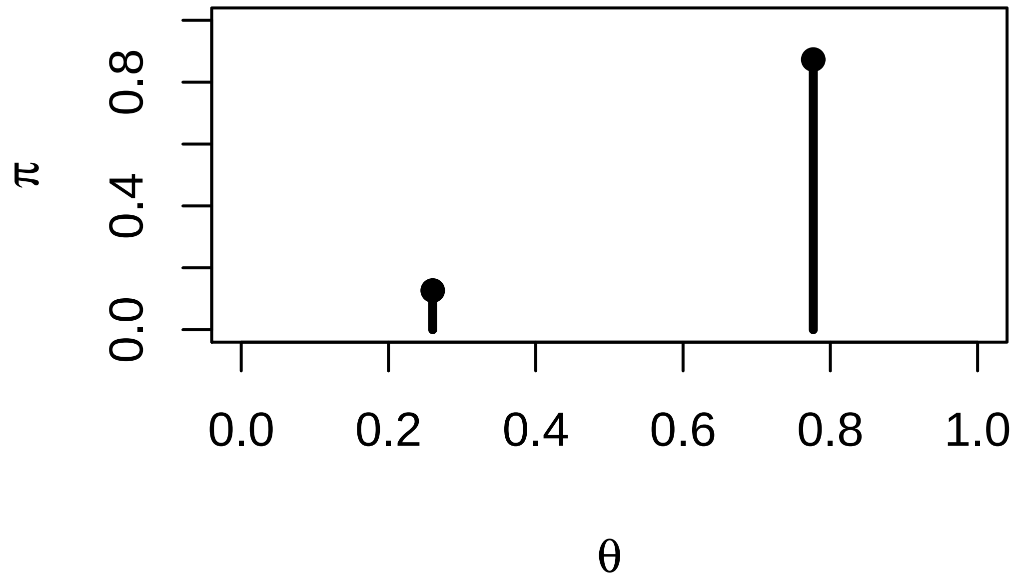
- N is identifiable **within** but not **between** model classes.
  e.g. finite vs. beta (infinite) mixture models for P

Modelling Heterogeneity by Finite Mixtures

1. **Individuals** – animals/errors differ in their catchability

Individuals belong to A latent classes:

- Membership probabilities $\{\pi_a\}$
- Capture probabilities $\{\phi_a\}$



$$f(p_i) = \sum_{a=1}^{A} \pi_a \delta(p_i - \phi_a) \qquad \sum_{a=1}^{A} \pi_a = 1$$

Modelling Heterogeneity by Finite Mixtures

2. **Samples** – probability of capture varies between samples
   - **Model** $M_{\mathrm{t[f]h}}$: k fixed effects

$$p_{ij} = \theta_{aj}$$
$$\mathrm{logit}(\theta_{aj}) = \mathrm{logit}(\phi_a) + \beta_j + \gamma_{aj}$$

Individual i is in latent class a

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Individual i is in latent class a

OR

   - **Model** M_{th} B latent classes: membership probabilities $\{\lambda_b\}$ capture probabilities $\{\psi_b\}$

$$p_{ij} = \theta_{ab}$$
$$\mathrm{logit}(\theta_{ab}) = \mathrm{logit}(\phi_a) + \mathrm{logit}(\psi_b) - \mathrm{logit}(\phi_1) + \gamma_{ab}$$

Individual i is in latent class a and
Sample j is in latent class b

Identifiability

- Ordering constraints on mixture component probabilities

$$0 < \phi_1 < \ldots < \phi_A < 1 \;\text{ and }\; \phi_1 = \psi_1 < \psi_2 < \ldots < \psi_B < 1$$

  Implemented by 'repulsive' priors (following Green 1995)

$$\underline{\phi}|A \sim \mathrm{EOS}(\mathrm{Uniform}(0,1)^{2A+1})$$

  $\{\phi_a\}$ are the even order statistics of $(2A+1)$ draws from $U(0,1)$
- Sum to zero constraints on fixed effects

$$\begin{aligned} \sum_j \beta_j &= \sum_a \gamma_{aj} = \sum_j \gamma_{aj} = 0 \quad \text{or} \\ \sum_a \gamma_{ab} &= \sum_b \gamma_{ab} = 0 \end{aligned}$$

  Ensured by degenerate Normal priors, e.g.

$$\beta_j \sim \mathrm{Normal}(0, {\sigma'_\beta}^2) \quad \text{with} \sum_j \beta_j = 0$$

Priors

			Models	
Parameter	Distribution	Constants	$M_{\text{t[f]h}}$	M_{th}
N	Geometric$(1-\eta)$	$\eta = 0.999$	$\star$	$\star$
A	Geometric$(1-\rho_h)$	$\rho_h = 0.8$	$\star$	$\star$
B	Geometric$(1-\rho_h)$	$\rho_h = 0.8$		$\star$
$\underline{\pi}\|A$	Dirichlet$(\underline{1}\alpha_h)$	$\alpha_h = \frac{3}{2}$	$\star$	$\star$
$\underline{\phi}\|A$	EOS$(\text{Uniform}(0,1)^{2A+1})$		$\star$	$\star$
σ^2_β	InverseGamma$(\nu_\beta, \kappa_\beta)$	$\nu_\beta = 3, \kappa_\beta = 40$	$\star$	
$\underline{\beta}$	DegenNormal$(k;0,\sigma^2_\beta)$		$\star$	
$\underline{\lambda}\|B$	Dirichlet$(\underline{1}\alpha_t)$	$\alpha_t = \frac{3}{2}$		$\star$
$\underline{\psi}\|B,\phi_1$	EOS$(\text{Uniform}(\phi_1,1)^{2B-1})$			$\star$
$\zeta\|A,(B)$	Bernoulli(p_γ)	$p_\gamma = \frac{1}{2}$	$\star$	$\star$
$\gamma\|\zeta,A,(B)$	DegenNormal$(A,B;0,\sigma^2_\gamma)$	$\sigma_\gamma = 5$	$\star$	$\star$
$\underline{b}\|B,\underline{\lambda}$	Discrete$(\{1,\ldots,B\};\underline{\lambda})$			$\star$

Bayesian Estimation – RJMCMC

Estimation by Reversible Jump MCMC

- Draw from posterior
$$p(\{m, \Theta_m\}|X)$$
- Reversible Jump MCMC allows dimension switching moves: Addition/deletion of mixture components.
- 4 types of MCMC move in model M_{h}.

RJMCMC Updates

MCMC Step	1				
	$\theta_1^{(1)}$ $\theta_2^{(1)}$ $\theta_3^{(1)}$				

RJMCMC Updates

MCMC Step	1	2			
	$\theta_1^{(1)}$ $\theta_2^{(1)}$ $\theta_3^{(1)}$	$\theta_1^{(2)}$ $\theta_2^{(2)}$ $\theta_3^{(2)}$			

RJMCMC Updates

MCMC Step	1	2	3		
	$\theta_1^{(1)}$ $\theta_2^{(1)}$ $\theta_3^{(1)}$	$\theta_1^{(2)}$ $\theta_2^{(2)}$ $\theta_3^{(2)}$	$\theta_1^{(3)}$ $\theta_2^{(3)}$ $\theta_3^{(3)}$		

RJMCMC Updates

MCMC Step	1	2	3	4	
	$\theta_1^{(1)}$ $\theta_2^{(1)}$ $\theta_3^{(1)}$	$\theta_1^{(2)}$ $\theta_2^{(2)}$ $\theta_3^{(2)}$	$\theta_1^{(3)}$ $\theta_2^{(3)}$ $\theta_3^{(3)}$	$\theta_1^{(4)}$ $\theta_2^{(4)}$ $\theta_3^{(4)}$	

RJMCMC Updates

MCMC Step	1	2	3	4	…
	$\theta_1^{(1)}$	$\theta_1^{(2)}$	$\theta_1^{(3)}$	$\theta_1^{(4)}$	…
	$\theta_2^{(1)}$	$\theta_2^{(2)}$	$\theta_2^{(3)}$	$\theta_2^{(4)}$	…
	$\theta_3^{(1)}$	$\theta_3^{(2)}$	$\theta_3^{(3)}$	$\theta_3^{(4)}$	…

RJMCMC Updates

MCMC Step	1	2	3	4	...
	$\theta_1^{(1)}$ $\theta_2^{(1)}$ $\theta_3^{(1)}$	$\theta_1^{(2)}$ $\theta_2^{(2)}$ $\theta_3^{(2)}$	$\theta_1^{(3)}$ $\theta_2^{(3)}$ $\theta_3^{(3)}$	$\theta_1^{(4)}$ $\theta_2^{(4)}$ $\theta_3^{(4)}$	... ...

RJMCMC Step	1				
	$m^{(1)}$ $\theta_{m^{(1)}:1}^{(1)}$ $\theta_{m^{(1)}:2}^{(1)}$				

RJMCMC Updates

MCMC Step	1	2	3	4	…
	$\theta_1^{(1)}$ $\theta_2^{(1)}$ $\theta_3^{(1)}$	$\theta_1^{(2)}$ $\theta_2^{(2)}$ $\theta_3^{(2)}$	$\theta_1^{(3)}$ $\theta_2^{(3)}$ $\theta_3^{(3)}$	$\theta_1^{(4)}$ $\theta_2^{(4)}$ $\theta_3^{(4)}$	… … …

RJMCMC Step	1	2			
	$m^{(1)}$ $\theta^{(1)}_{m^{(1)}:1}$ $\theta^{(1)}_{m^{(1)}:2}$	$m^{(2)}$ $\theta^{(2)}_{m^{(2)}:1}$ $\theta^{(2)}_{m^{(2)}:2}$ $\theta^{(2)}_{m^{(2)}:3}$ $\theta^{(2)}_{m^{(2)}:4}$			

RJMCMC Updates

MCMC Step	1	2	3	4	...
	$\theta_1^{(1)}$ $\theta_2^{(1)}$ $\theta_3^{(1)}$	$\theta_1^{(2)}$ $\theta_2^{(2)}$ $\theta_3^{(2)}$	$\theta_1^{(3)}$ $\theta_2^{(3)}$ $\theta_3^{(3)}$	$\theta_1^{(4)}$ $\theta_2^{(4)}$ $\theta_3^{(4)}$	... ...

RJMCMC Step	1	2	3		
	$m^{(1)}$ $\theta^{(1)}_{m^{(1)}:1}$ $\theta^{(1)}_{m^{(1)}:2}$	$m^{(2)}$ $\theta^{(2)}_{m^{(2)}:1}$ $\theta^{(2)}_{m^{(2)}:2}$ $\theta^{(2)}_{m^{(2)}:3}$ $\theta^{(2)}_{m^{(2)}:4}$	$m^{(3)}$ $\theta^{(3)}_{m^{(3)}:1}$ $\theta^{(3)}_{m^{(3)}:2}$ $\theta^{(3)}_{m^{(3)}:3}$ $\theta^{(3)}_{m^{(3)}:4}$		

RJMCMC Updates

MCMC Step	1	2	3	4	...
	$\theta_1^{(1)}$ $\theta_2^{(1)}$ $\theta_3^{(1)}$	$\theta_1^{(2)}$ $\theta_2^{(2)}$ $\theta_3^{(2)}$	$\theta_1^{(3)}$ $\theta_2^{(3)}$ $\theta_3^{(3)}$	$\theta_1^{(4)}$ $\theta_2^{(4)}$ $\theta_3^{(4)}$	... ...

RJMCMC Step	1	2	3	4	
	$m^{(1)}$ $\theta^{(1)}_{m^{(1)}:1}$ $\theta^{(1)}_{m^{(1)}:2}$	$m^{(2)}$ $\theta^{(2)}_{m^{(2)}:1}$ $\theta^{(2)}_{m^{(2)}:2}$ $\theta^{(2)}_{m^{(2)}:3}$ $\theta^{(2)}_{m^{(2)}:4}$	$m^{(3)}$ $\theta^{(3)}_{m^{(3)}:1}$ $\theta^{(3)}_{m^{(3)}:2}$ $\theta^{(3)}_{m^{(3)}:3}$ $\theta^{(3)}_{m^{(3)}:4}$	$m^{(4)}$ $\theta^{(4)}_{m^{(4)}:1}$ $\theta^{(4)}_{m^{(4)}:2}$	

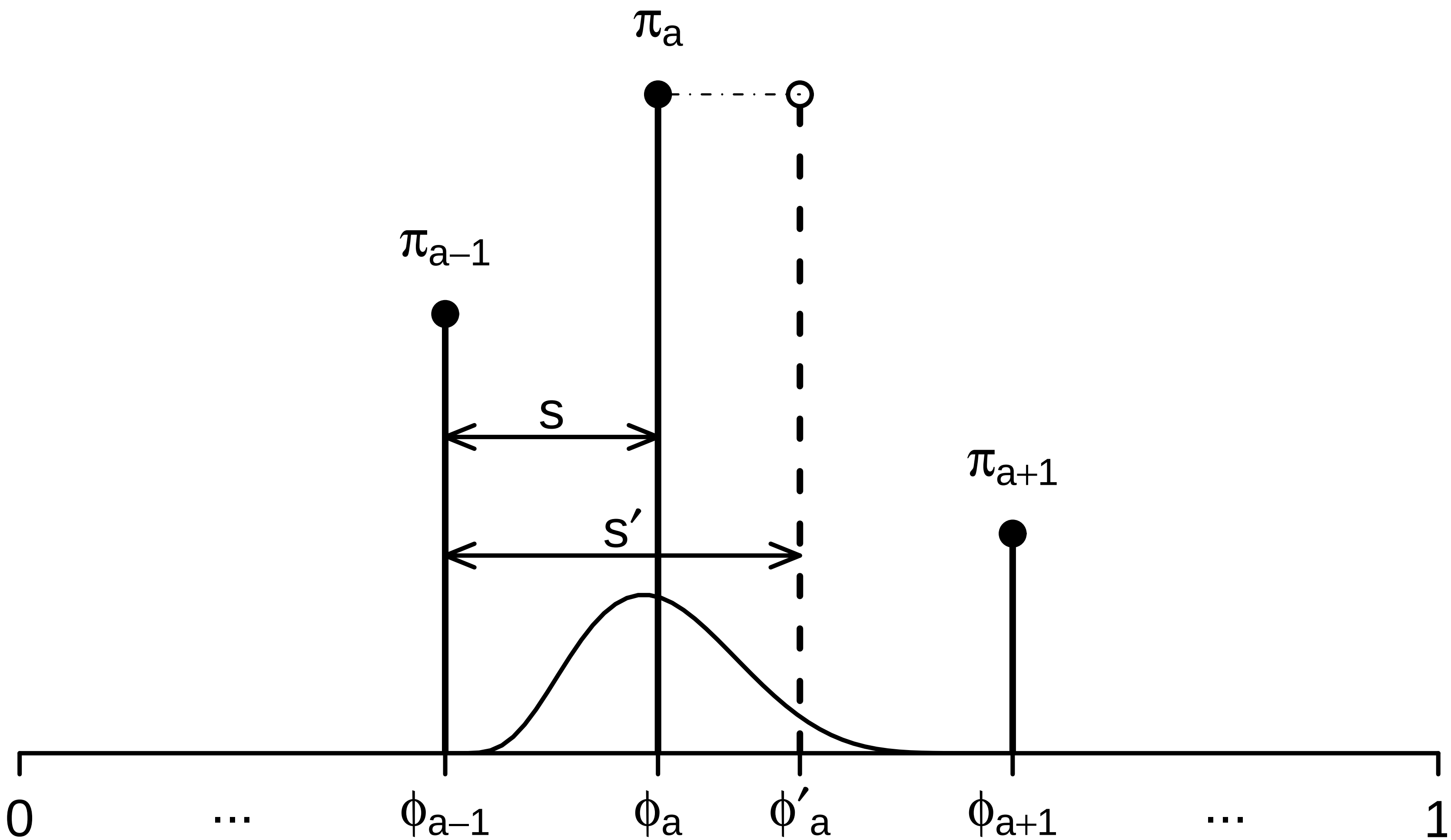
RJMCMC Updates

MCMC Step	1	2	3	4	...
	$\theta_1^{(1)}$	$\theta_1^{(2)}$	$\theta_1^{(3)}$	$\theta_1^{(4)}$	...
	$\theta_2^{(1)}$	$\theta_2^{(2)}$	$\theta_2^{(3)}$	$\theta_2^{(4)}$	...
	$\theta_3^{(1)}$	$\theta_3^{(2)}$	$\theta_3^{(3)}$	$\theta_3^{(4)}$	...

RJMCMC Step	1	2	3	4	...
	$m^{(1)}$	$m^{(2)}$	$m^{(3)}$	$m^{(4)}$	...
	$\theta^{(1)}_{m^{(1)}:1}$	$\theta^{(2)}_{m^{(2)}:1}$	$\theta^{(3)}_{m^{(3)}:1}$	$\theta^{(4)}_{m^{(4)}:1}$	...
	$\theta^{(1)}_{m^{(1)}:2}$	$\theta^{(2)}_{m^{(2)}:2}$	$\theta^{(3)}_{m^{(3)}:2}$	$\theta^{(4)}_{m^{(4)}:2}$	...
		$\theta^{(2)}_{m^{(2)}:3}$	$\theta^{(3)}_{m^{(3)}:3}$		...
		$\theta^{(2)}_{m^{(2)}:4}$	$\theta^{(3)}_{m^{(3)}:4}$		...

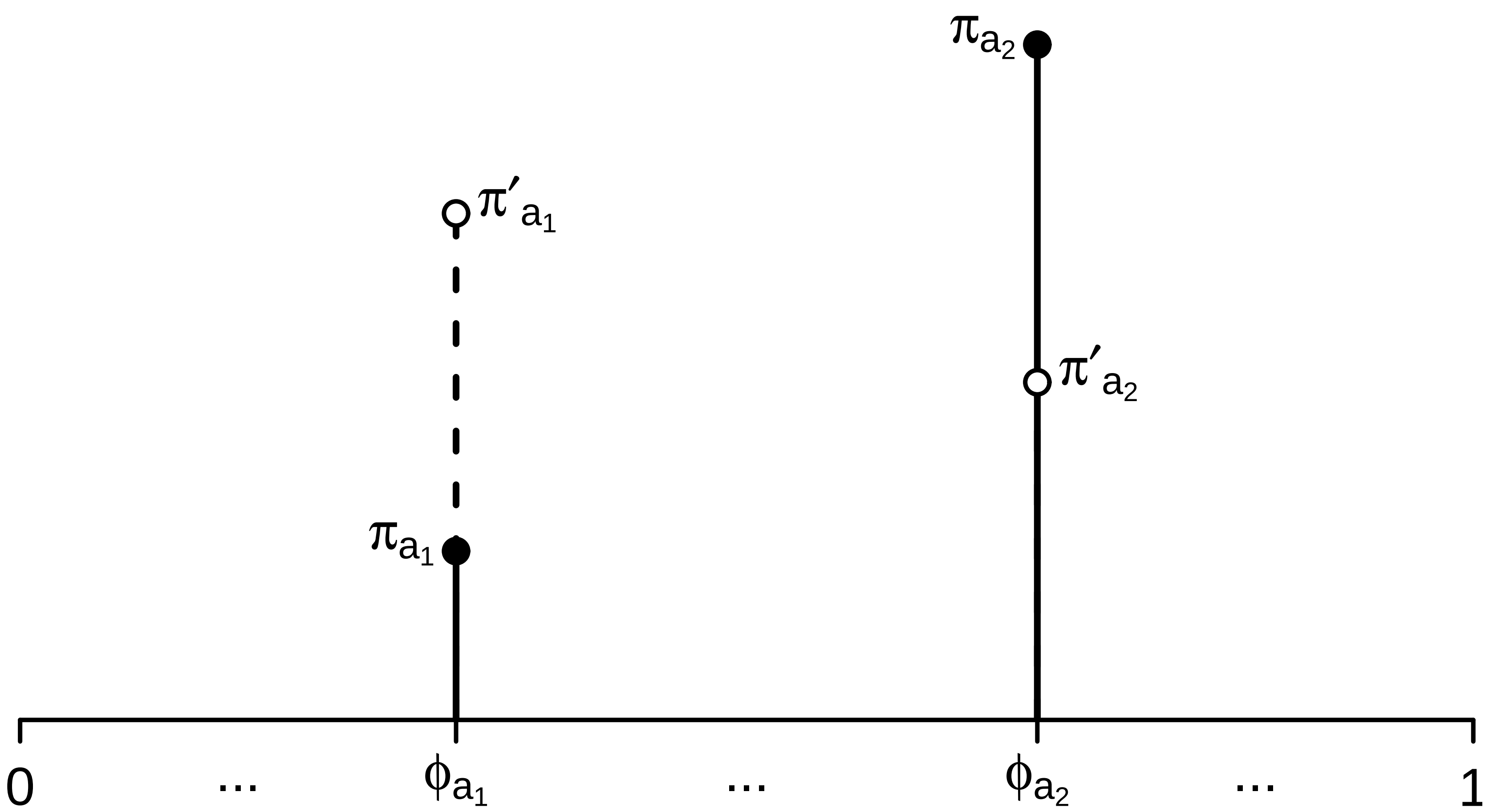
RJMCMC Updates

(1) Shift a support point



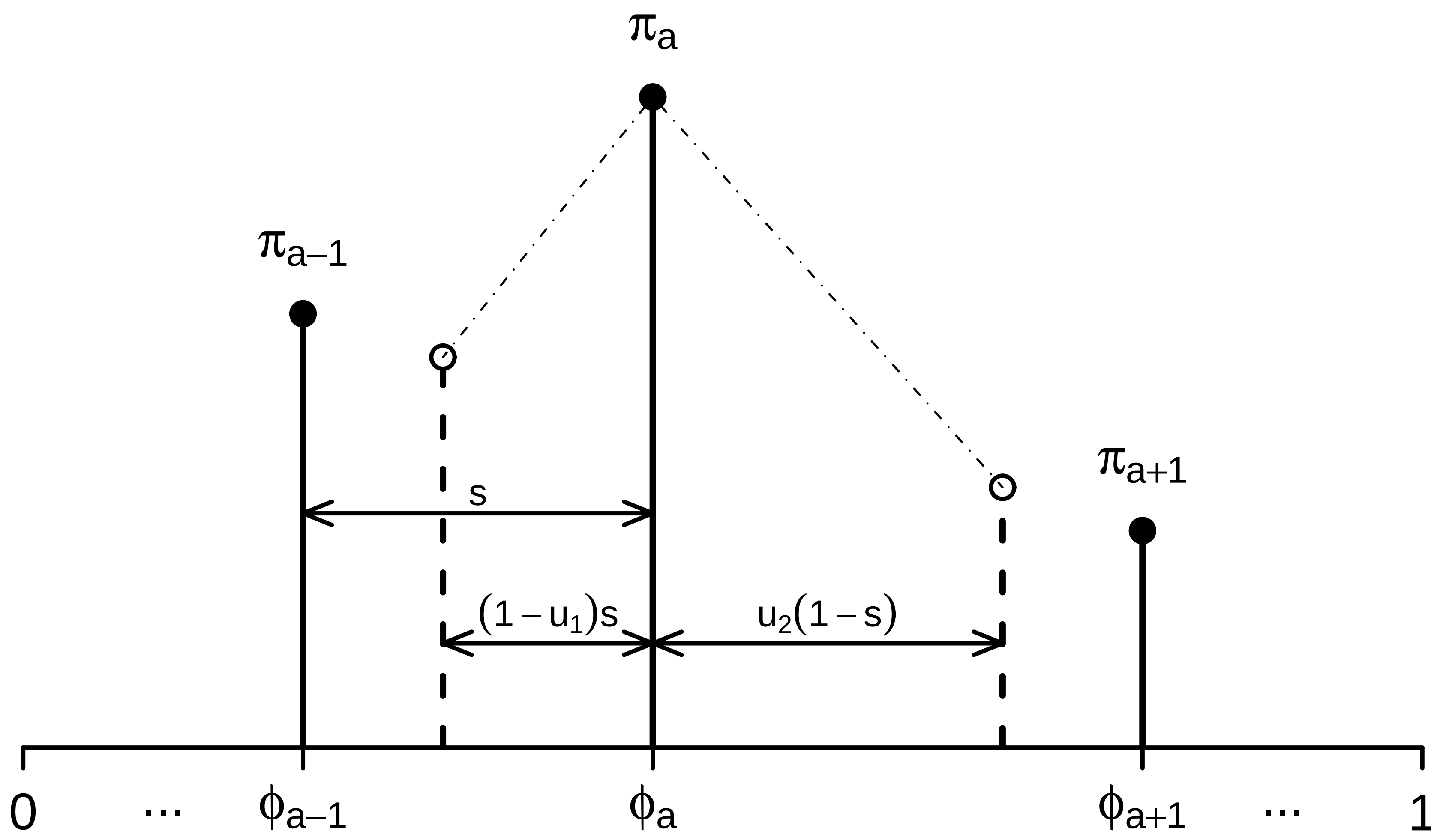
RJMCMC Updates

(2) Exchange probability between points



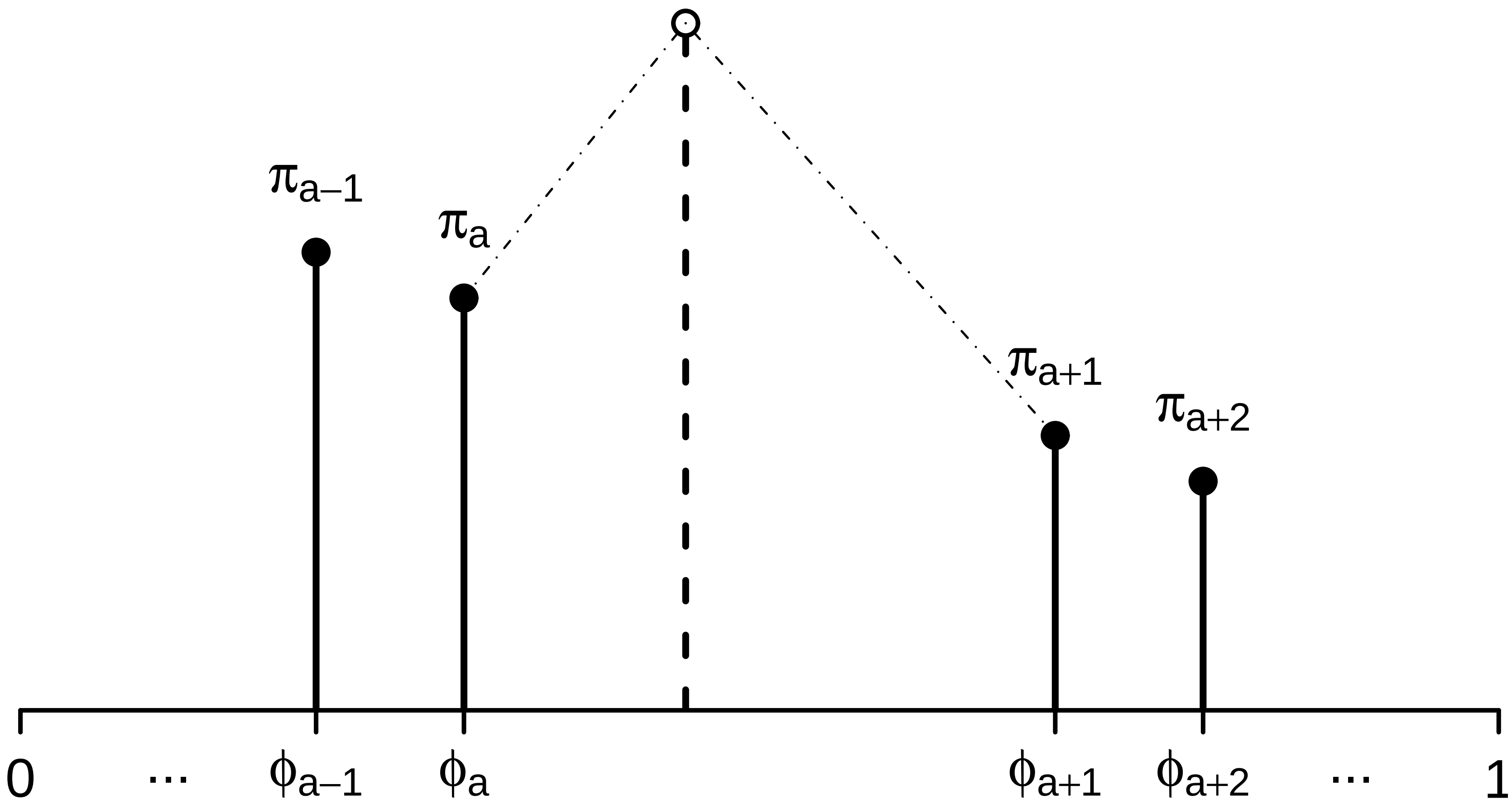
RJMCMC Updates

(3) Split a support point



RJMCMC Updates

(4) Merge two support points



Label switching and poor mixing

Estimation of finite mixture models affected if the MCMC sampler **can't mix** because of

- artificial identifiability constraints,
  (e.g. so that individuals/samples become persistently misallocated) or
- updating protocols making one mode inaccessible from another.

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- artificial identifiability constraints,
  (e.g. so that individuals/samples become persistently misallocated) or
- updating protocols making one mode inaccessible from another.

In our case:

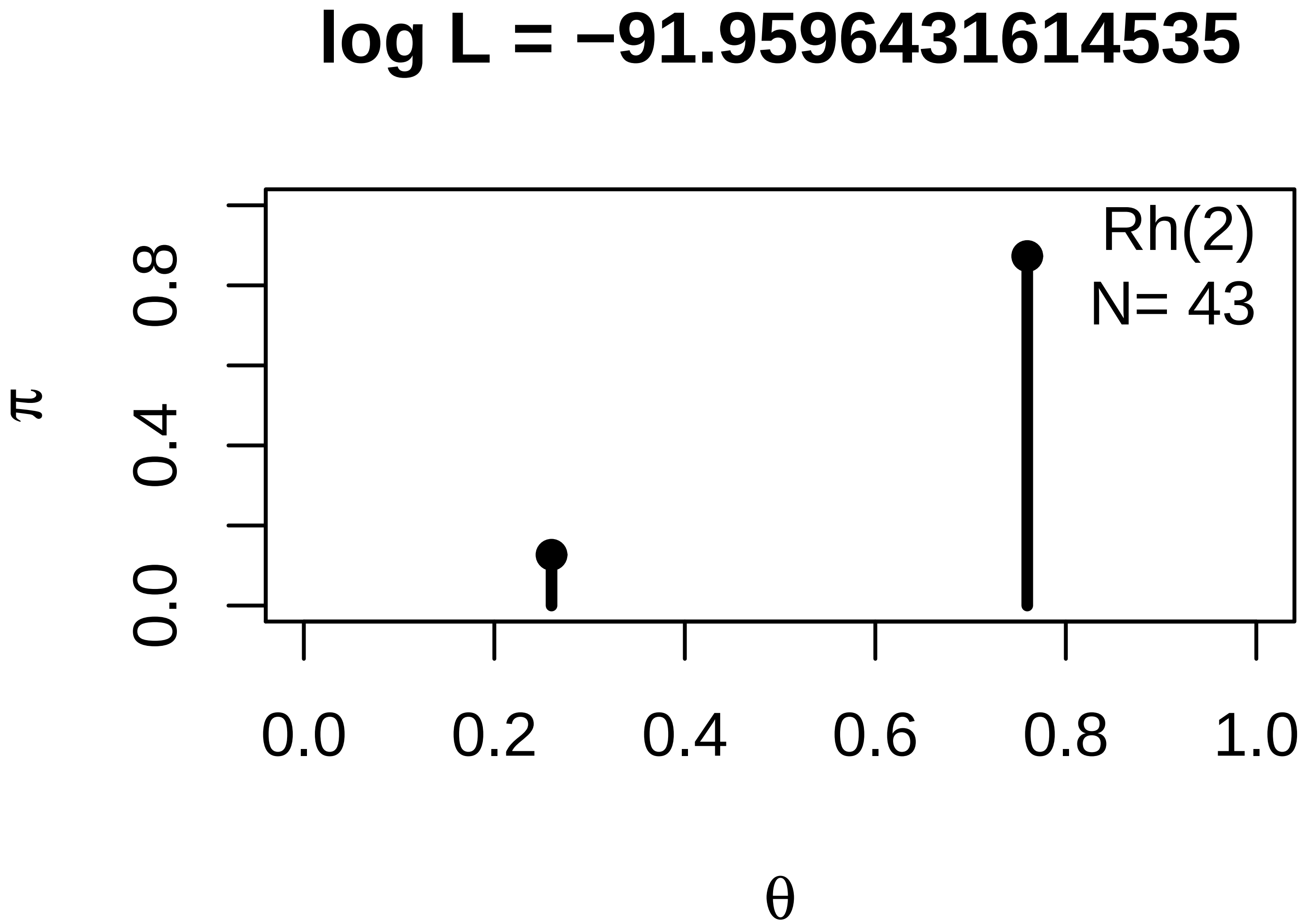
- Mixture components are labelled by a **single parameter**:
  have a natural ordering (e.g. preventing misallocations);
- Dimension-switching RJMCMC allows components to be deleted and reappear – the chain moves rapidly around model space.

Application: AT&T Switch Testing

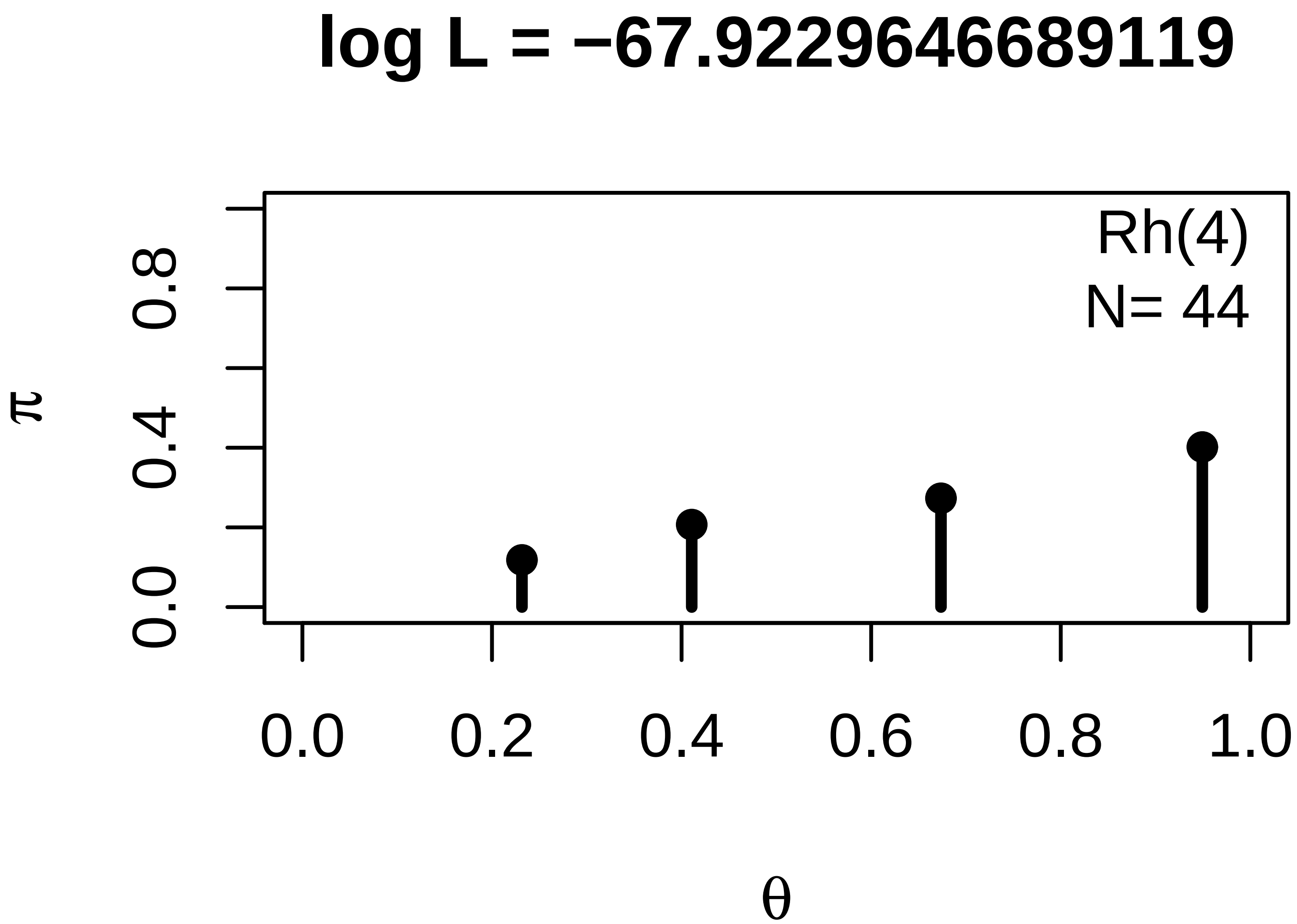
$k = 6$ reviewers tested switches and found $D = 43$ errors (Basu 1998)

Error	Reviewer j						
i	1	2	3	4	5	6	x_i
1	1	0	0	0	0	0	1
2	0	0	0	1	0	1	2
3	0	0	0	0	1	0	1
4	1	0	0	0	0	0	1
5	0	0	0	1	0	0	1
6	0	0	0	1	0	0	1
7	0	0	0	1	0	0	1
8	1	0	0	0	0	0	1
9	1	0	0	0	0	0	1
10	0	0	0	1	0	0	1
11	1	0	0	1	0	0	2
12	1	0	0	0	0	0	1
13	1	0	0	1	0	0	2
14	1	0	0	0	0	0	1
15	1	0	0	1	0	0	2
16	1	0	0	1	0	0	2
17	1	1	0	1	1	1	5
18	1	0	0	0	0	1	2
19	0	1	0	0	0	0	1
20	0	1	0	0	0	0	1
21	1	0	0	1	0	0	2
22	1	0	0	1	0	0	2
23	0	0	0	0	0	1	1
24	0	0	0	0	1	0	1
25	0	0	0	1	1	0	2
26	1	0	0	0	0	0	1
27	0	0	0	0	1	0	1
28	0	0	1	0	0	0	1
29	1	0	0	0	0	0	1
30	1	0	0	1	1	0	3
31	1	0	0	1	0	0	2
32	1	0	0	0	0	0	1
33	1	0	0	0	0	0	1
34	0	0	1	0	0	0	1
35	0	0	1	0	0	0	1
36	0	0	0	0	0	1	1
37	0	0	1	0	0	0	1
38	1	0	0	0	0	1	2
39	1	0	0	0	0	0	1
40	1	0	0	0	0	0	1
41	0	0	0	0	1	0	1
42	1	0	0	0	0	0	1
43	1	0	0	0	0	0	1
n_j	25	3	4	15	7	6	60

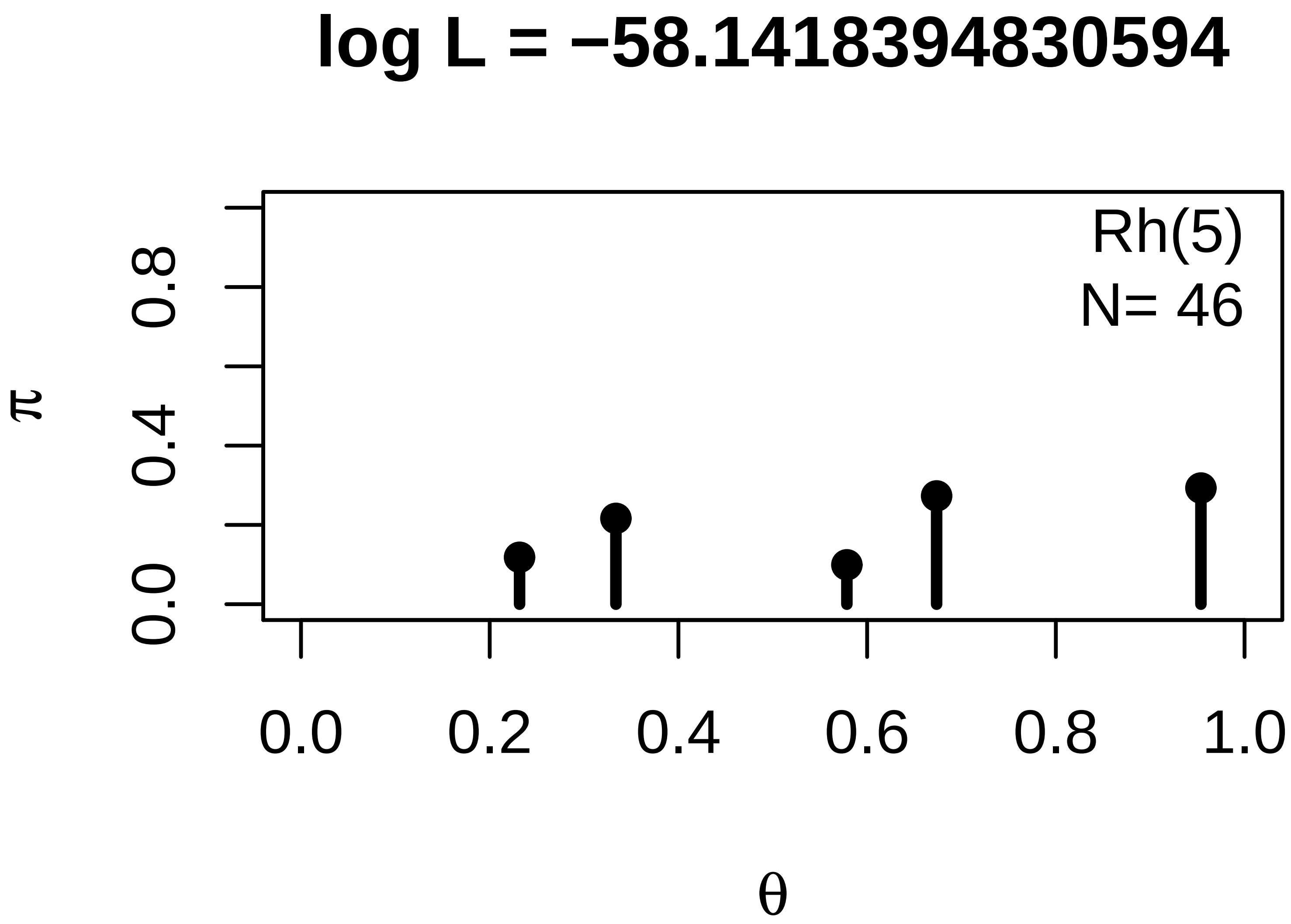
Step 1



Step 2



Step 3



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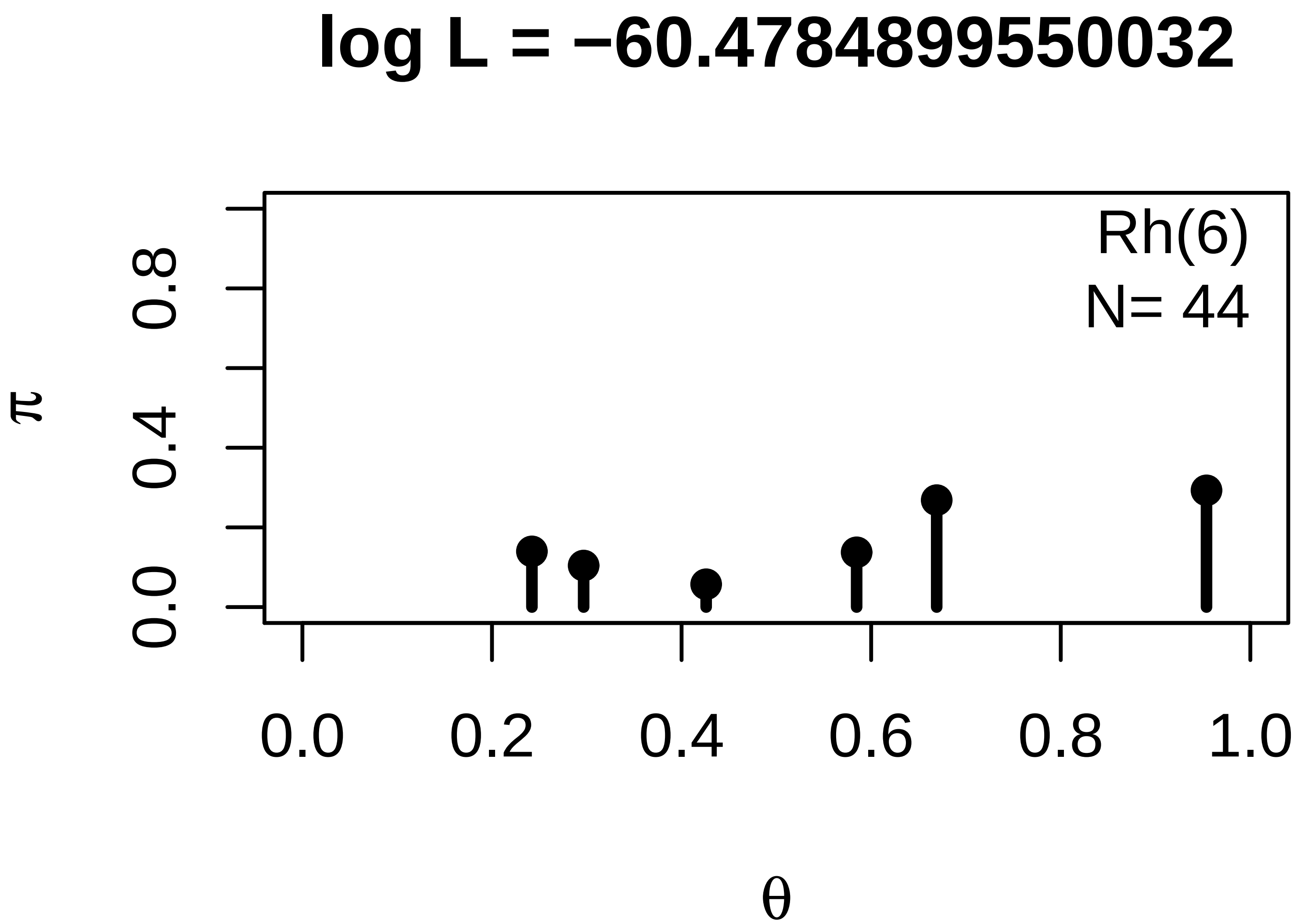

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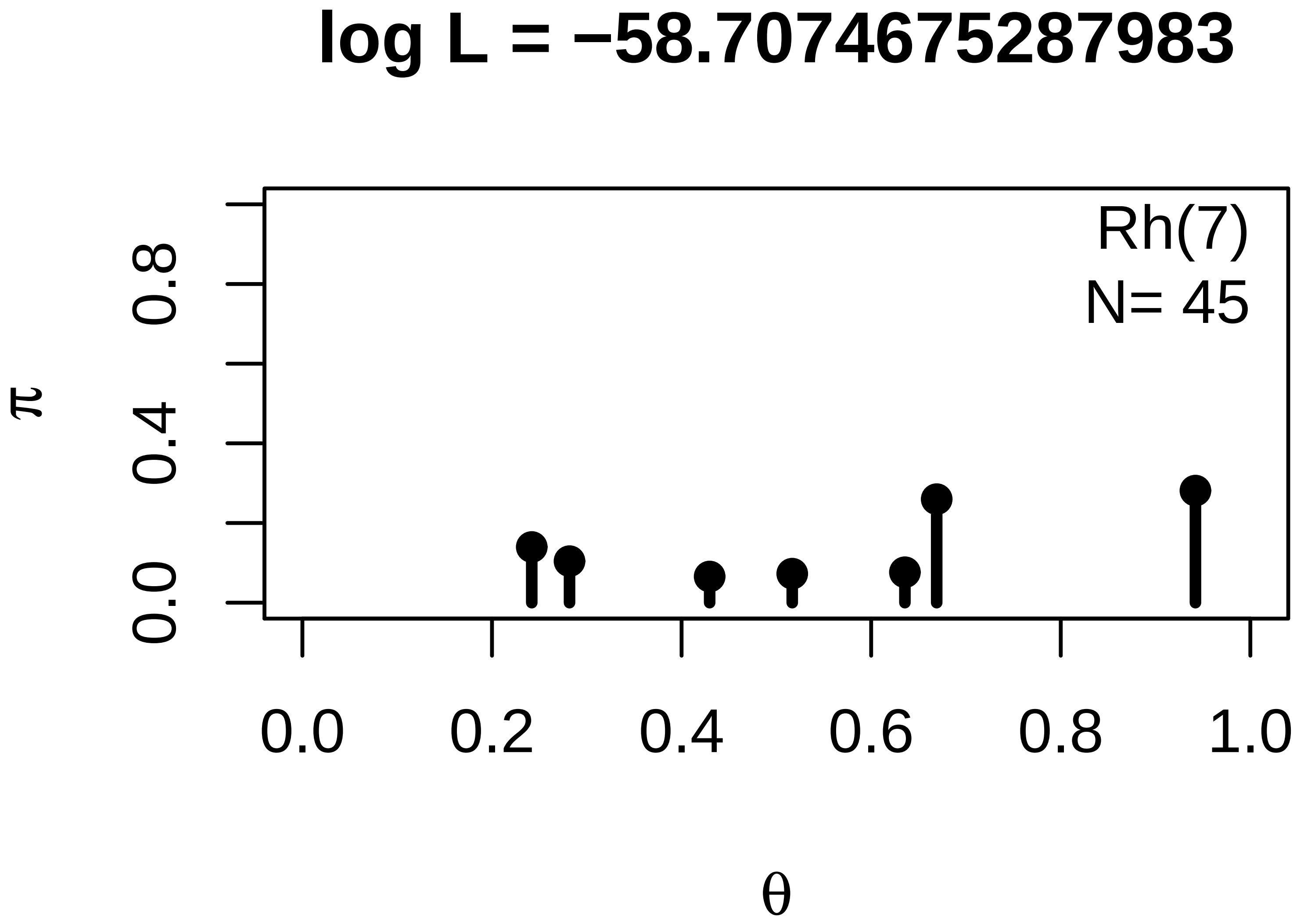

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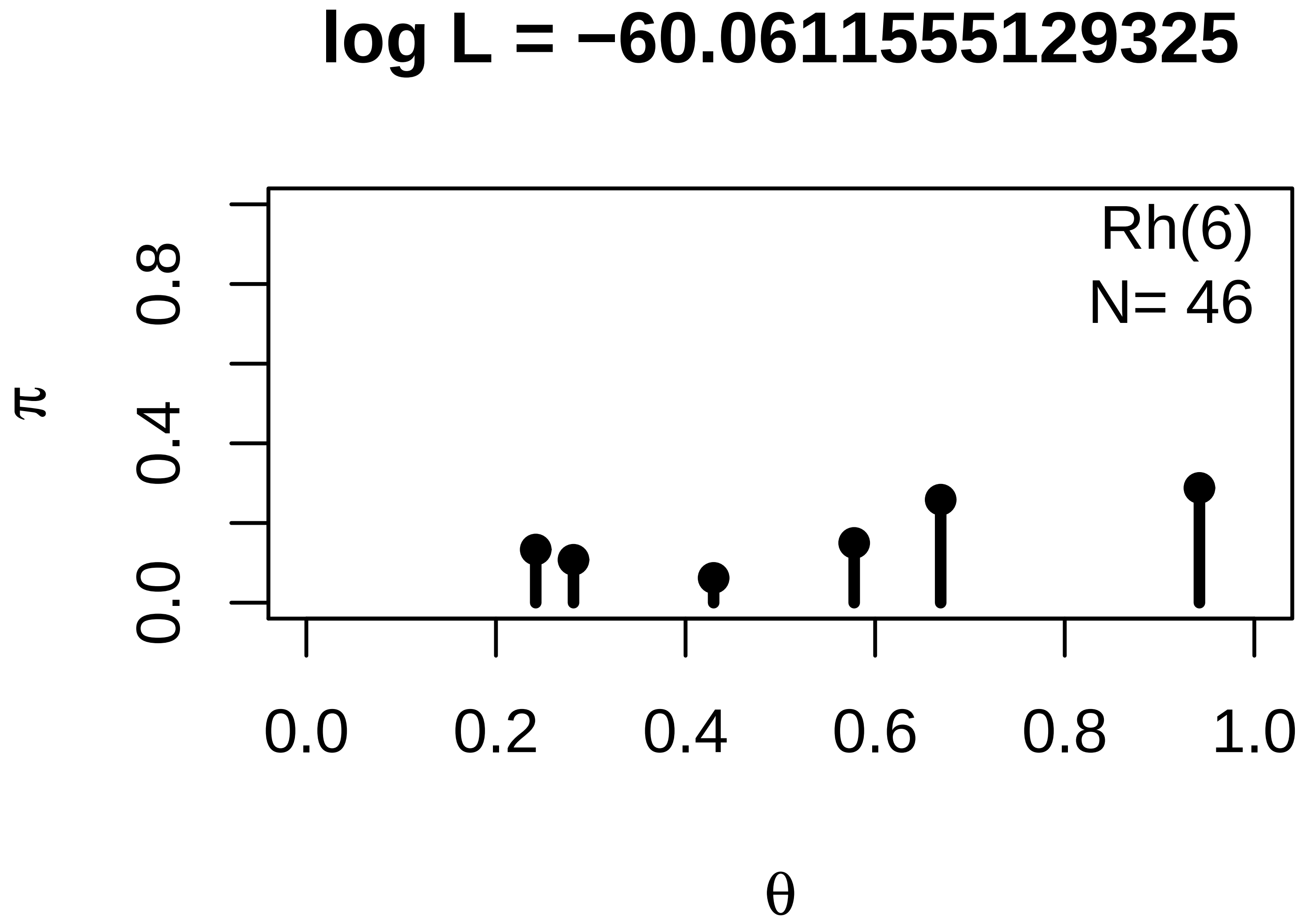

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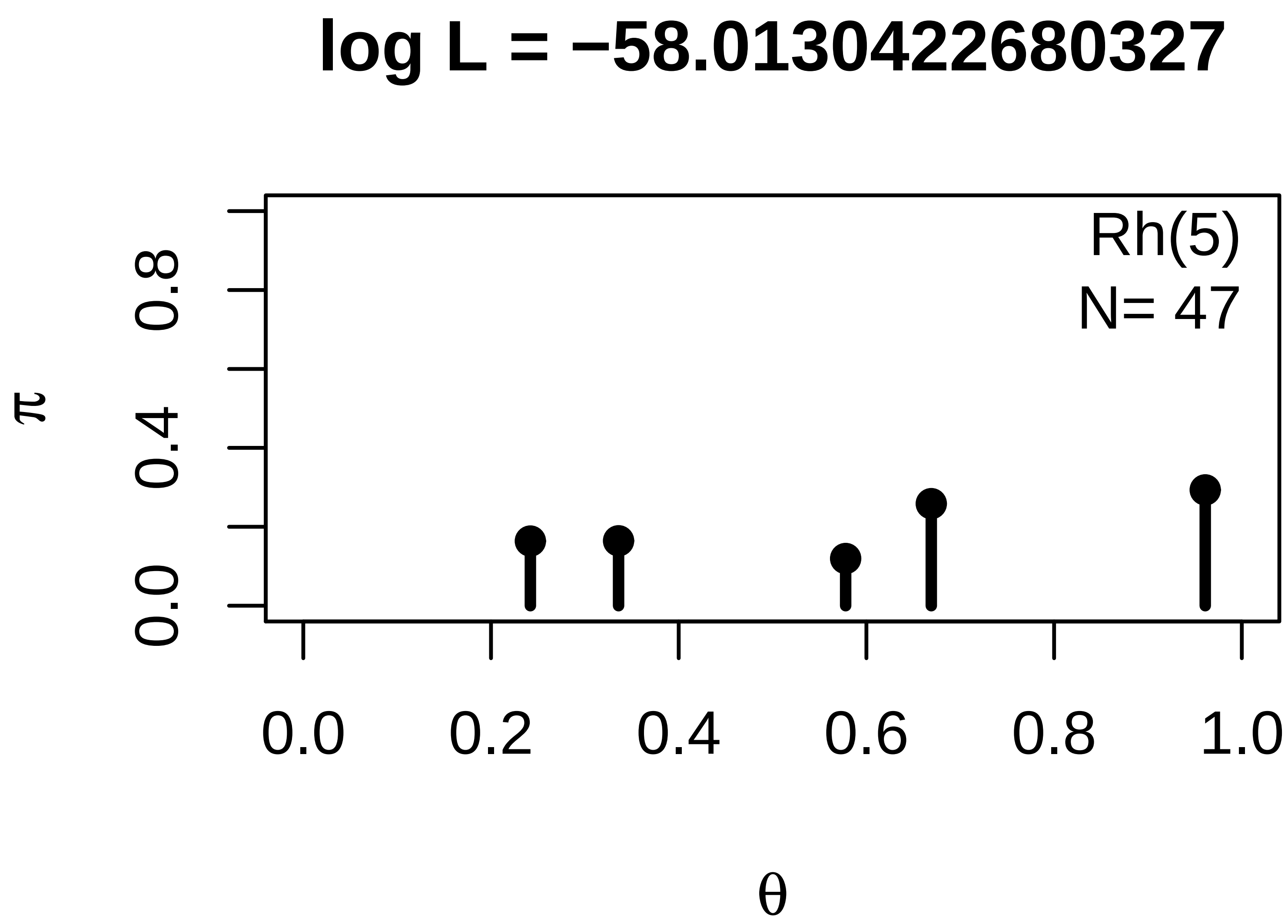
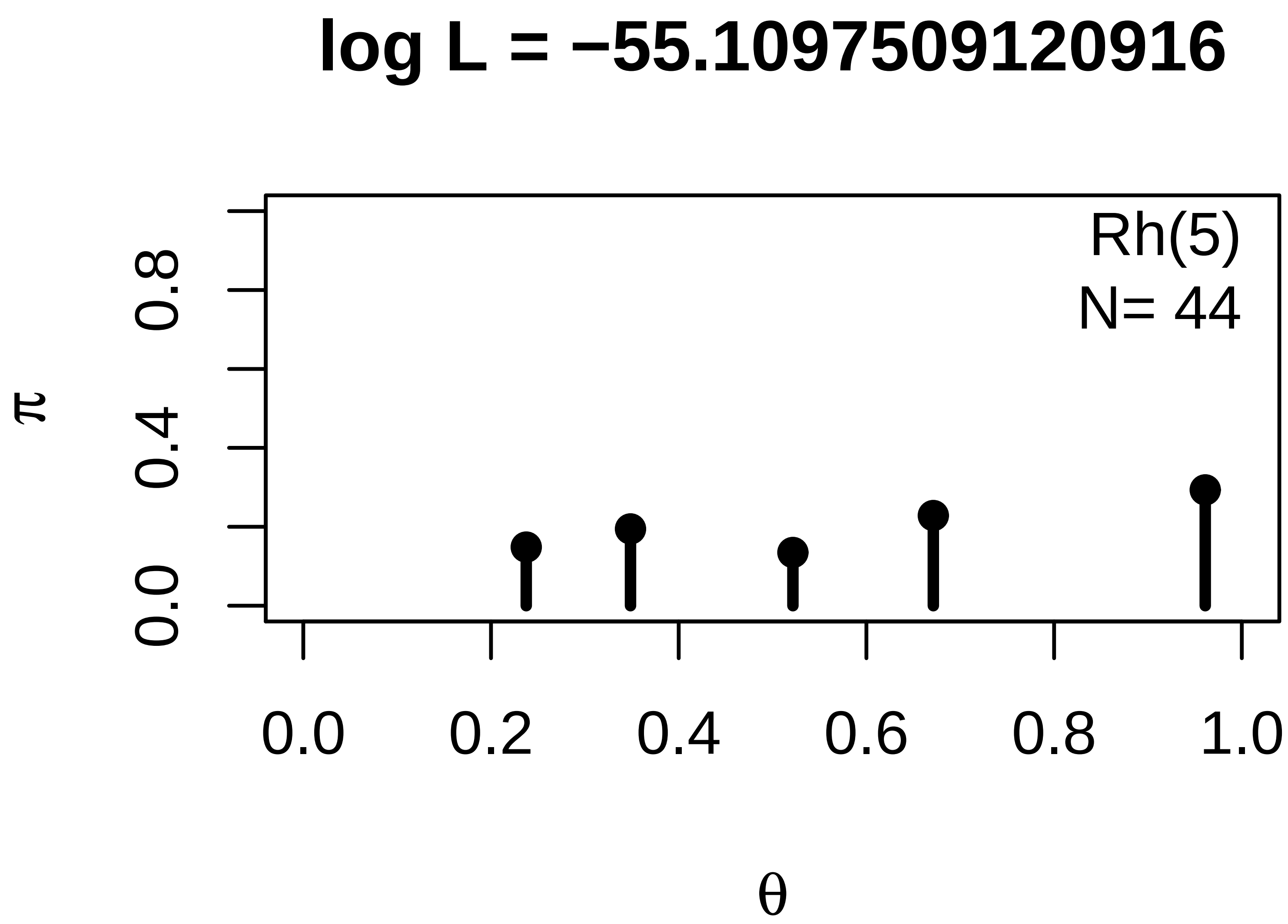


Step 8



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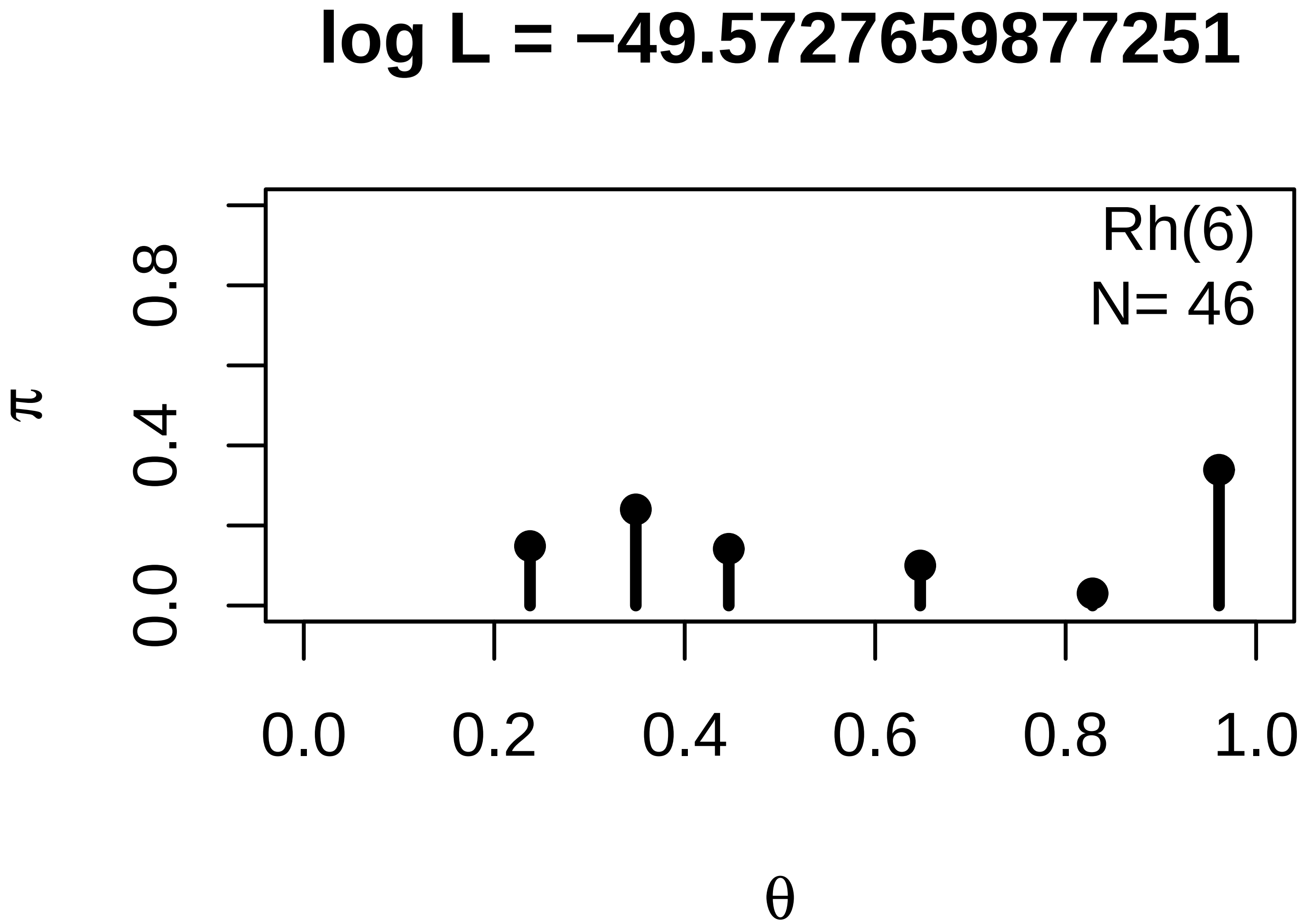
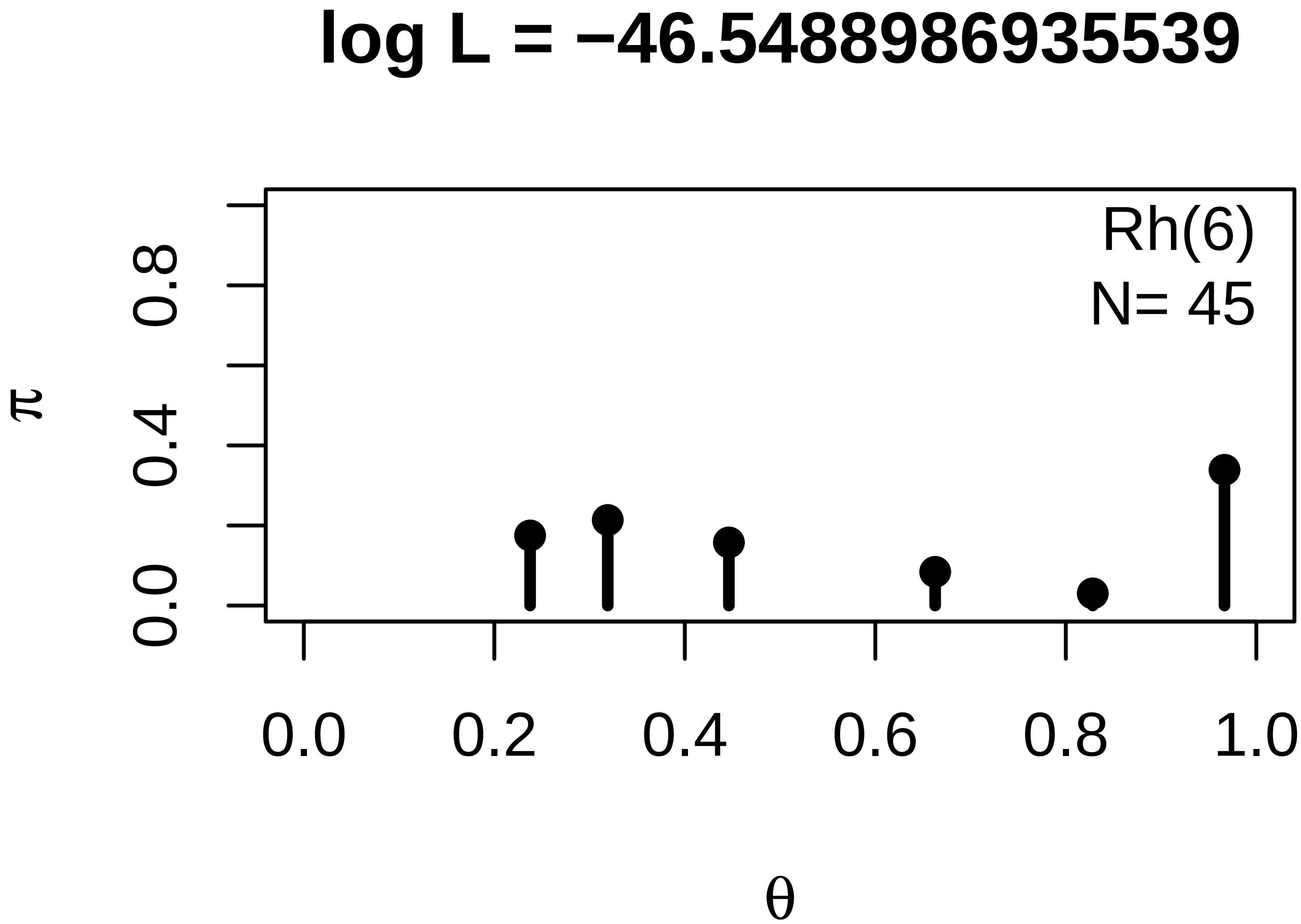


Step 10



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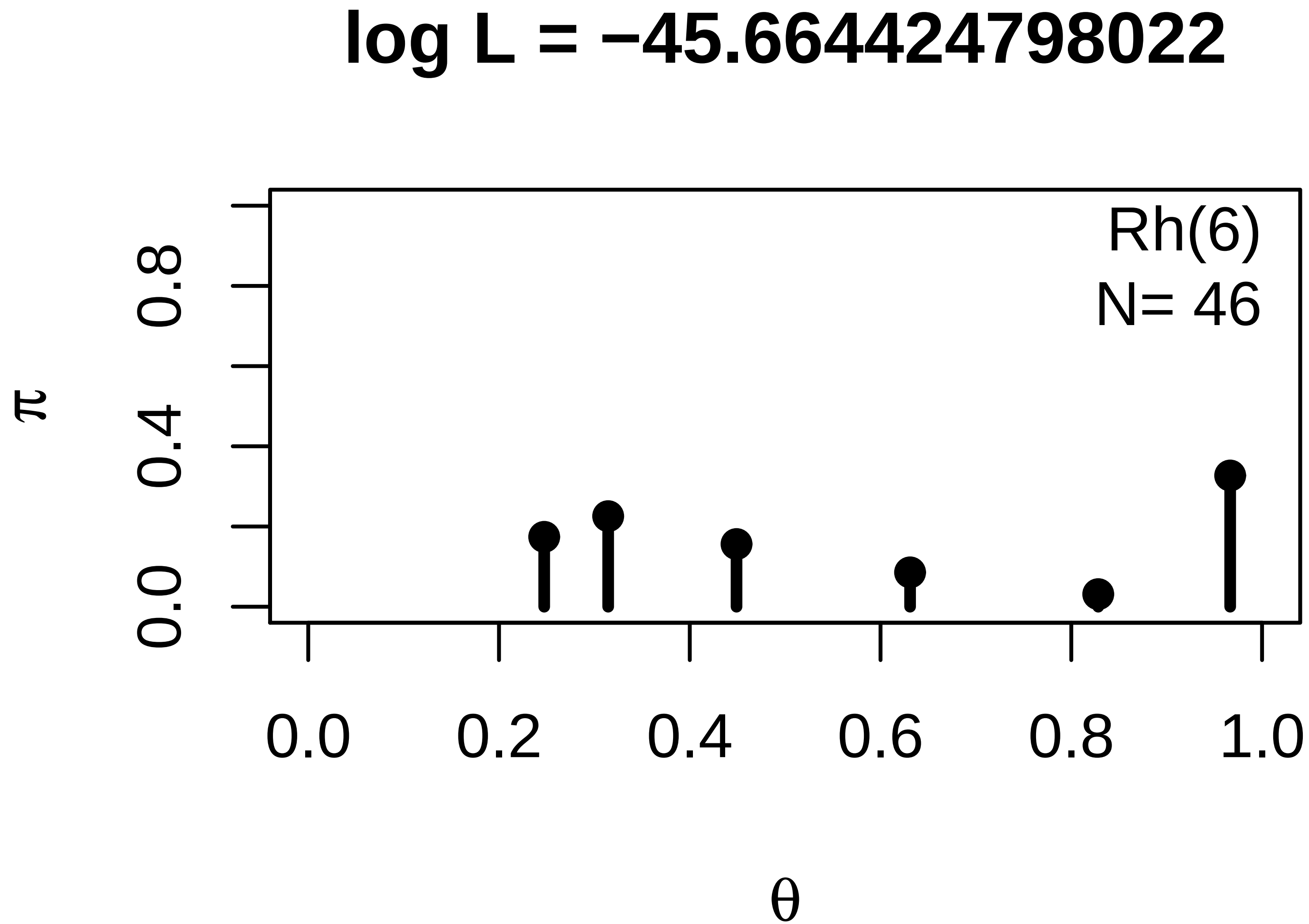
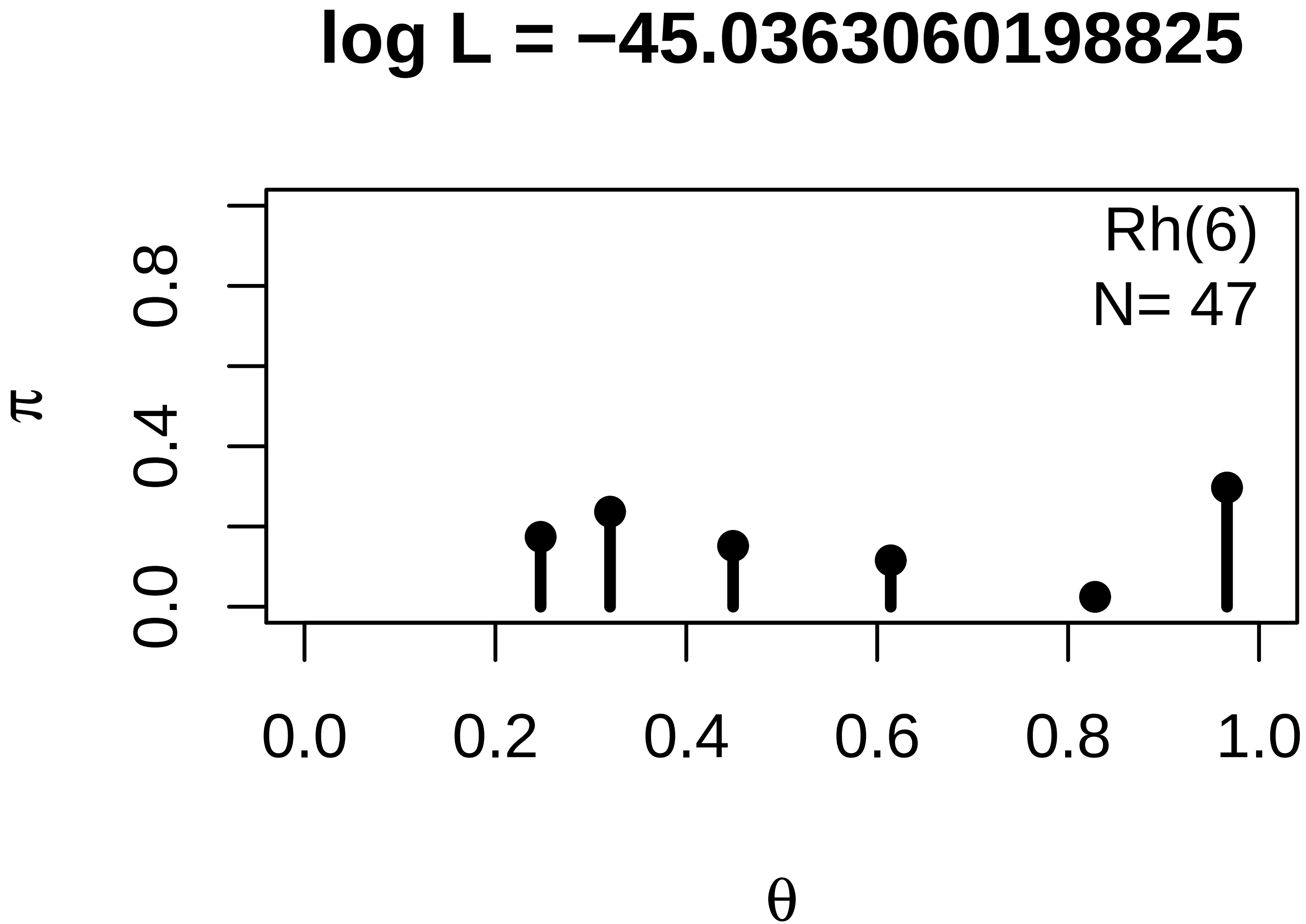
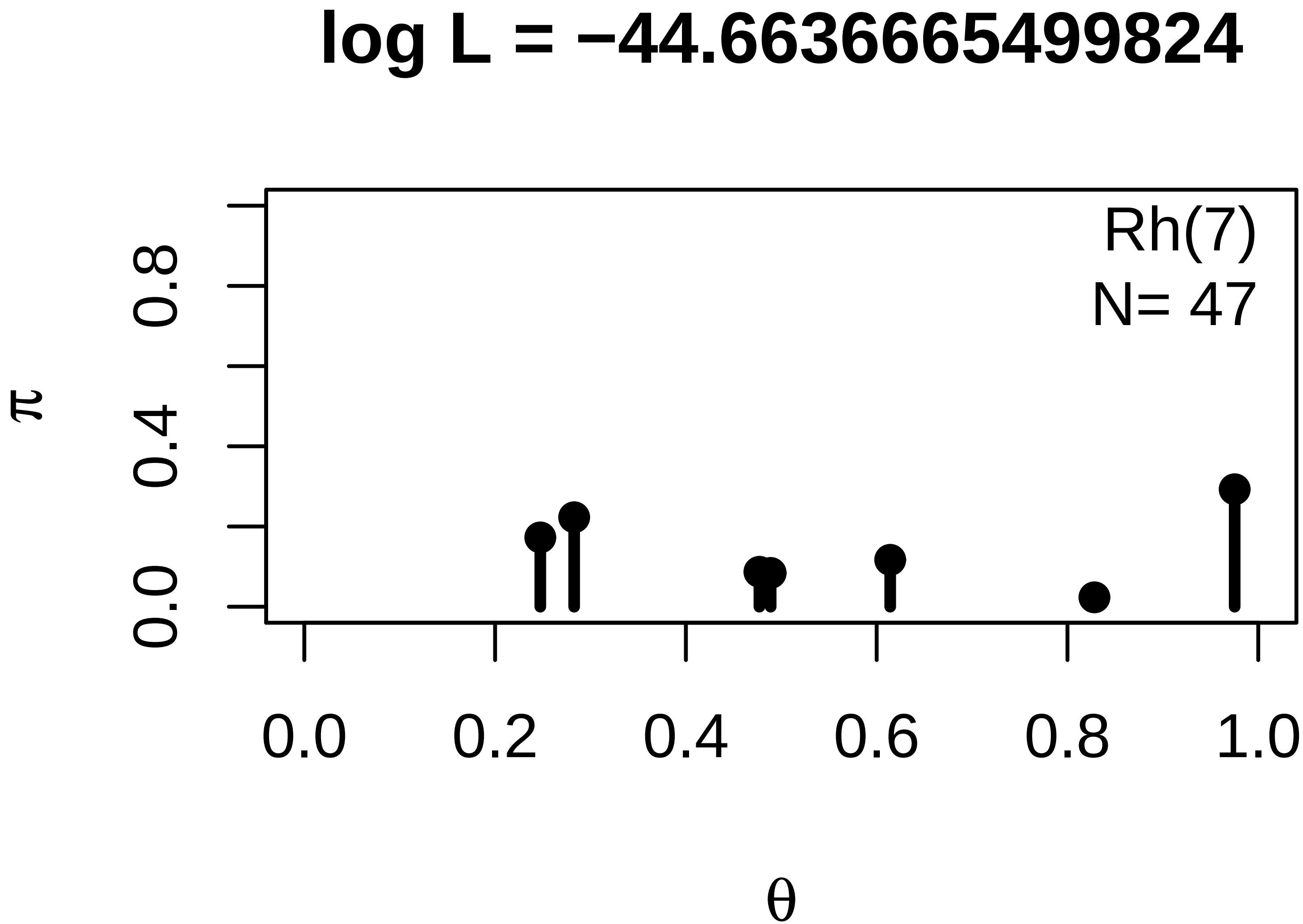


Step 12



Step 13



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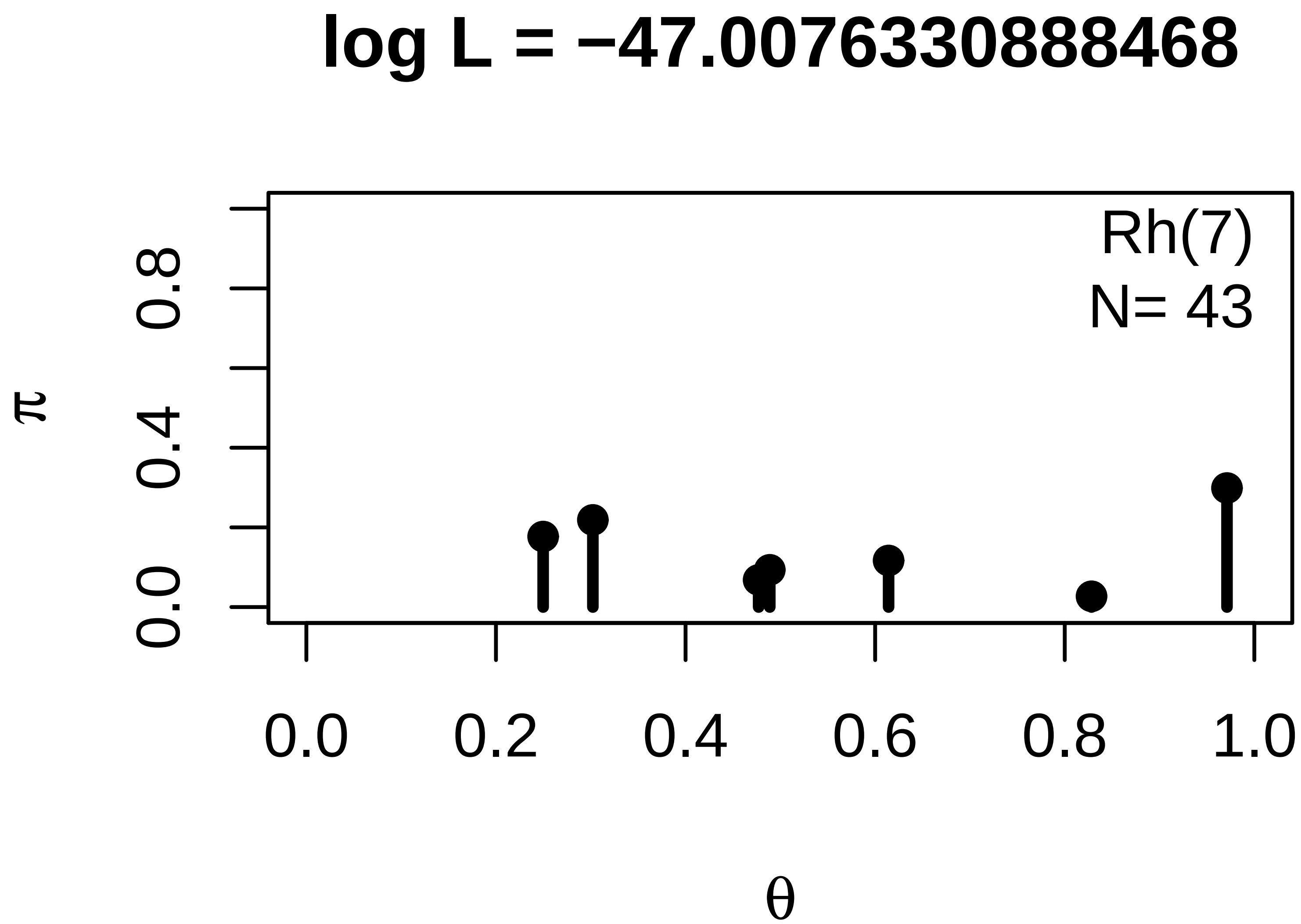
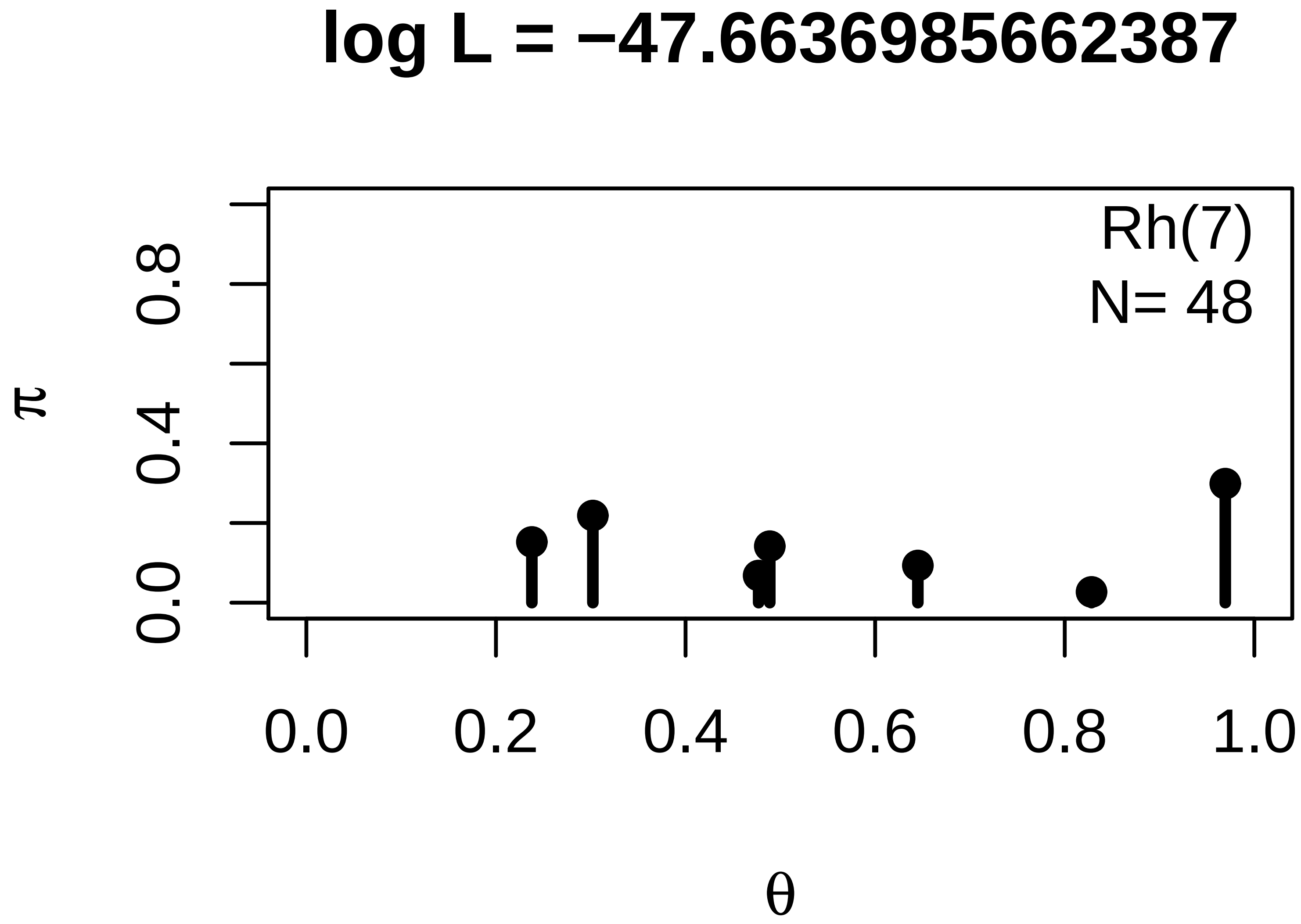
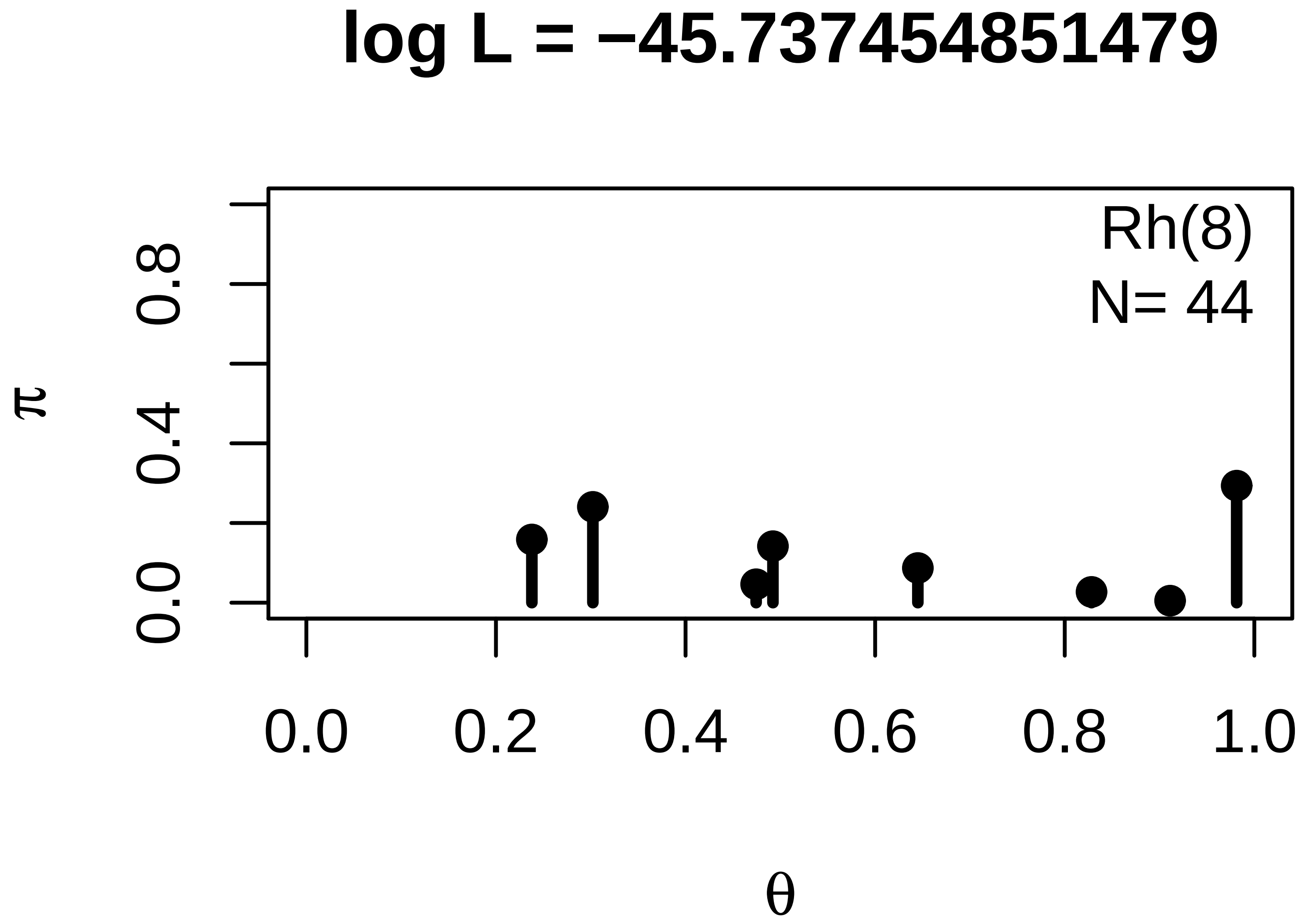


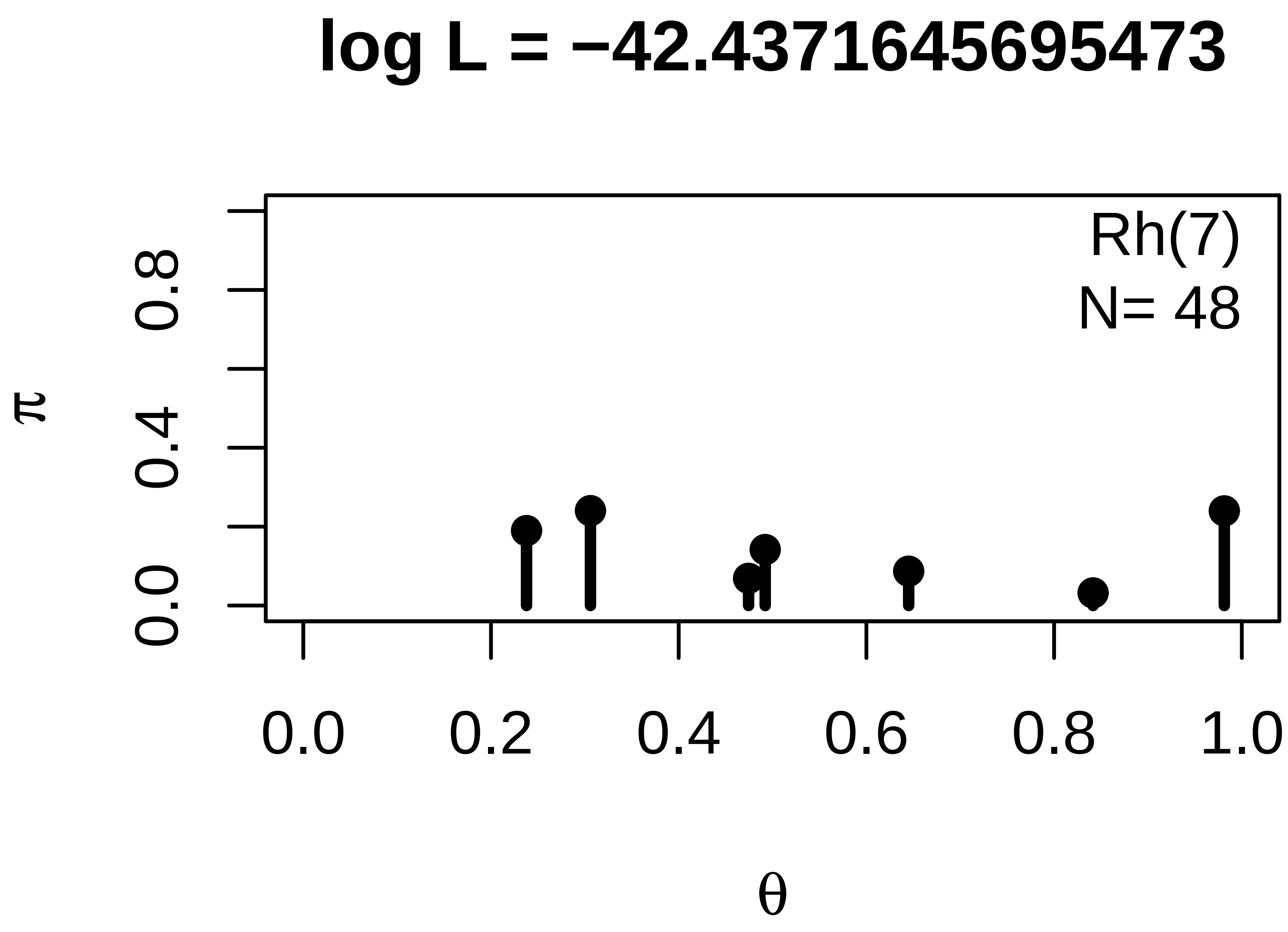
Step 15



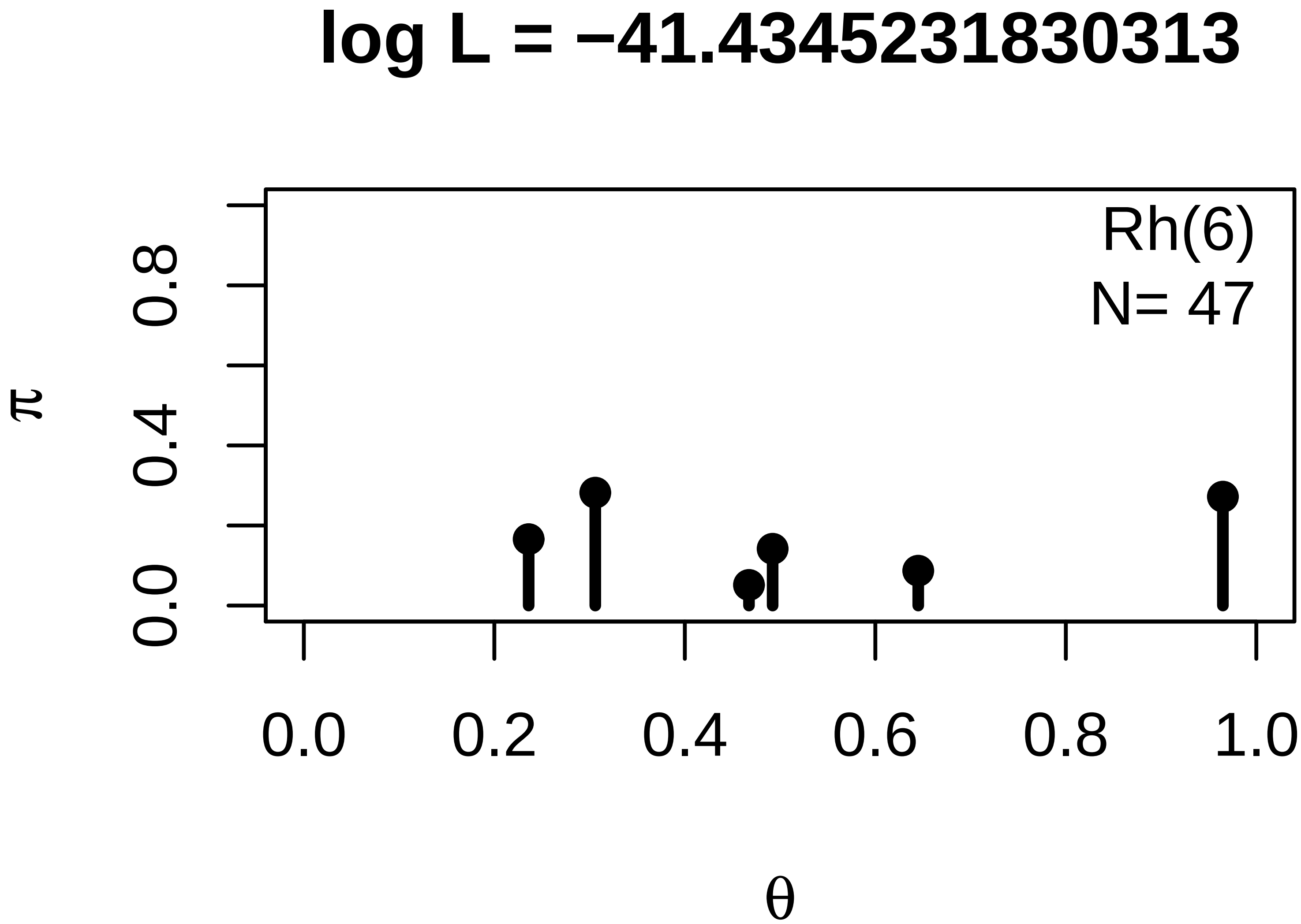
Step 16



Step 17



Step 18



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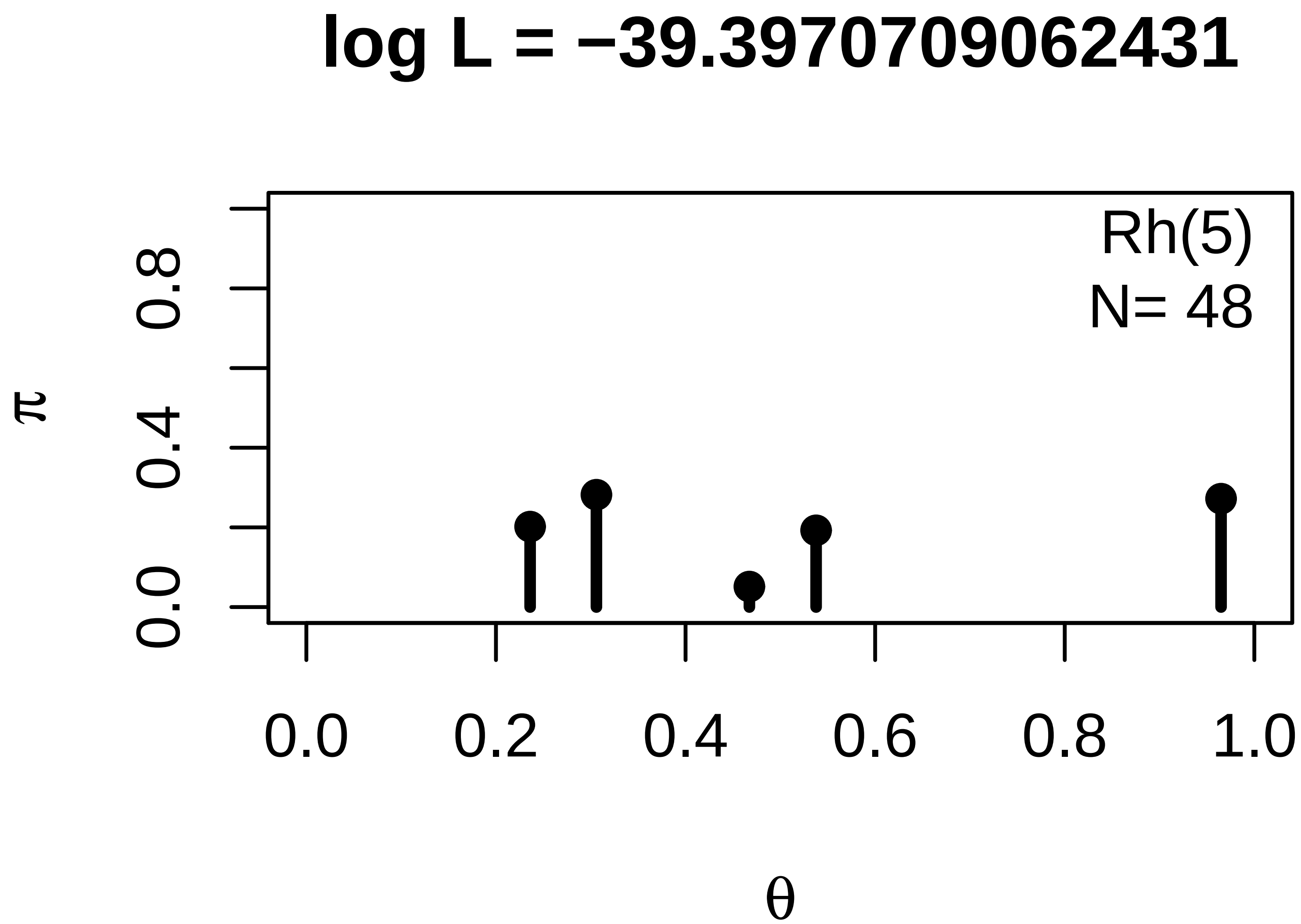

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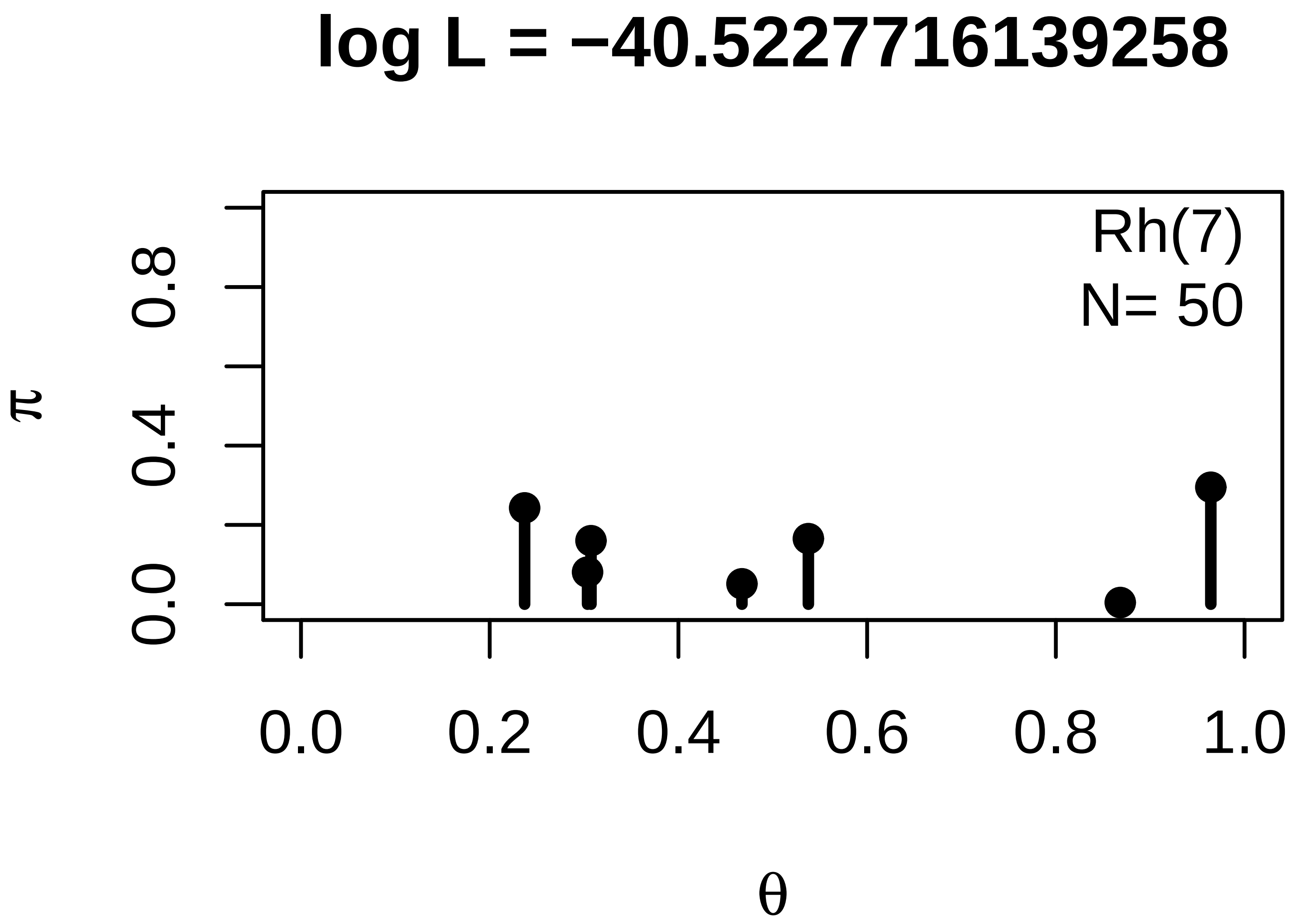

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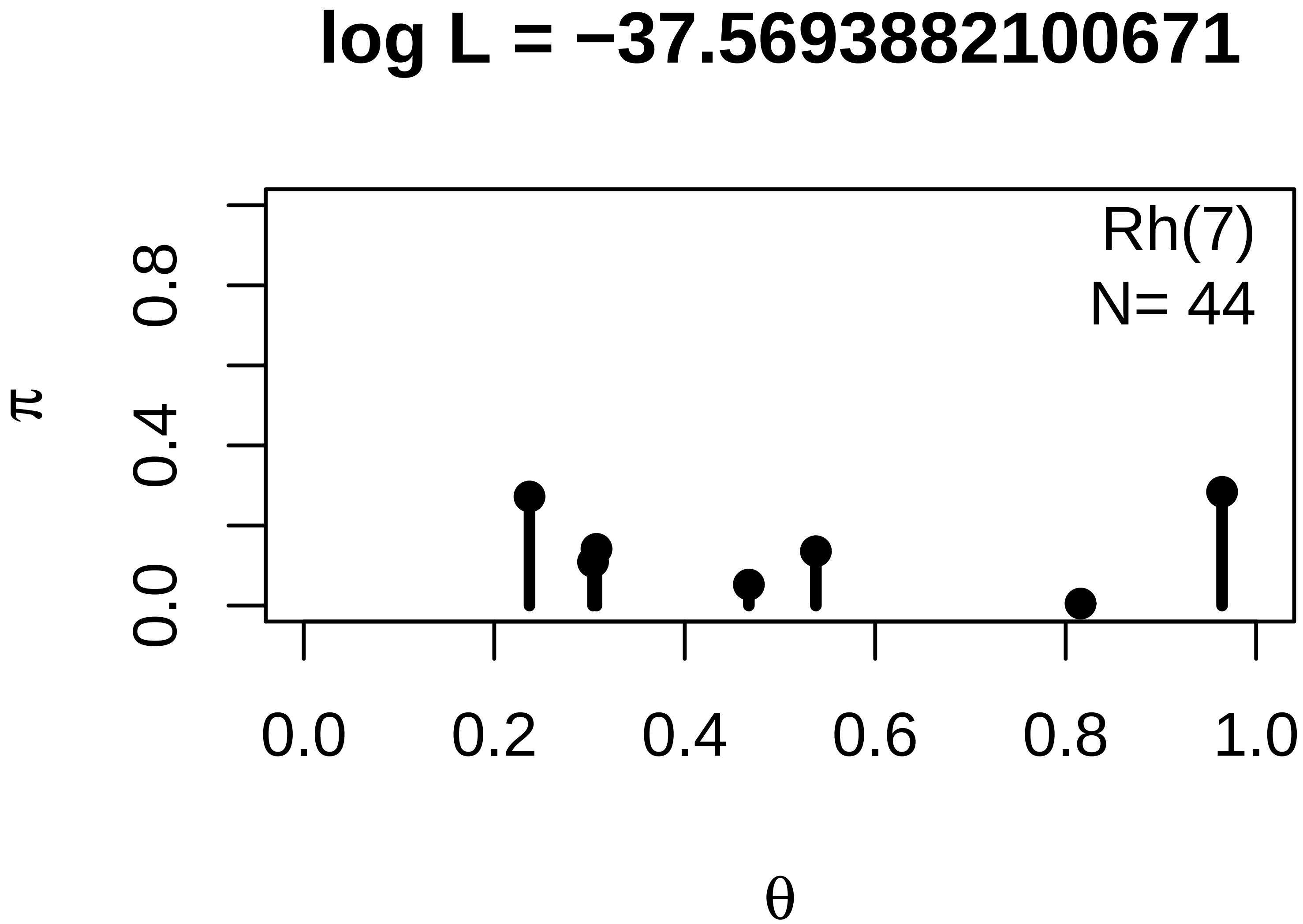
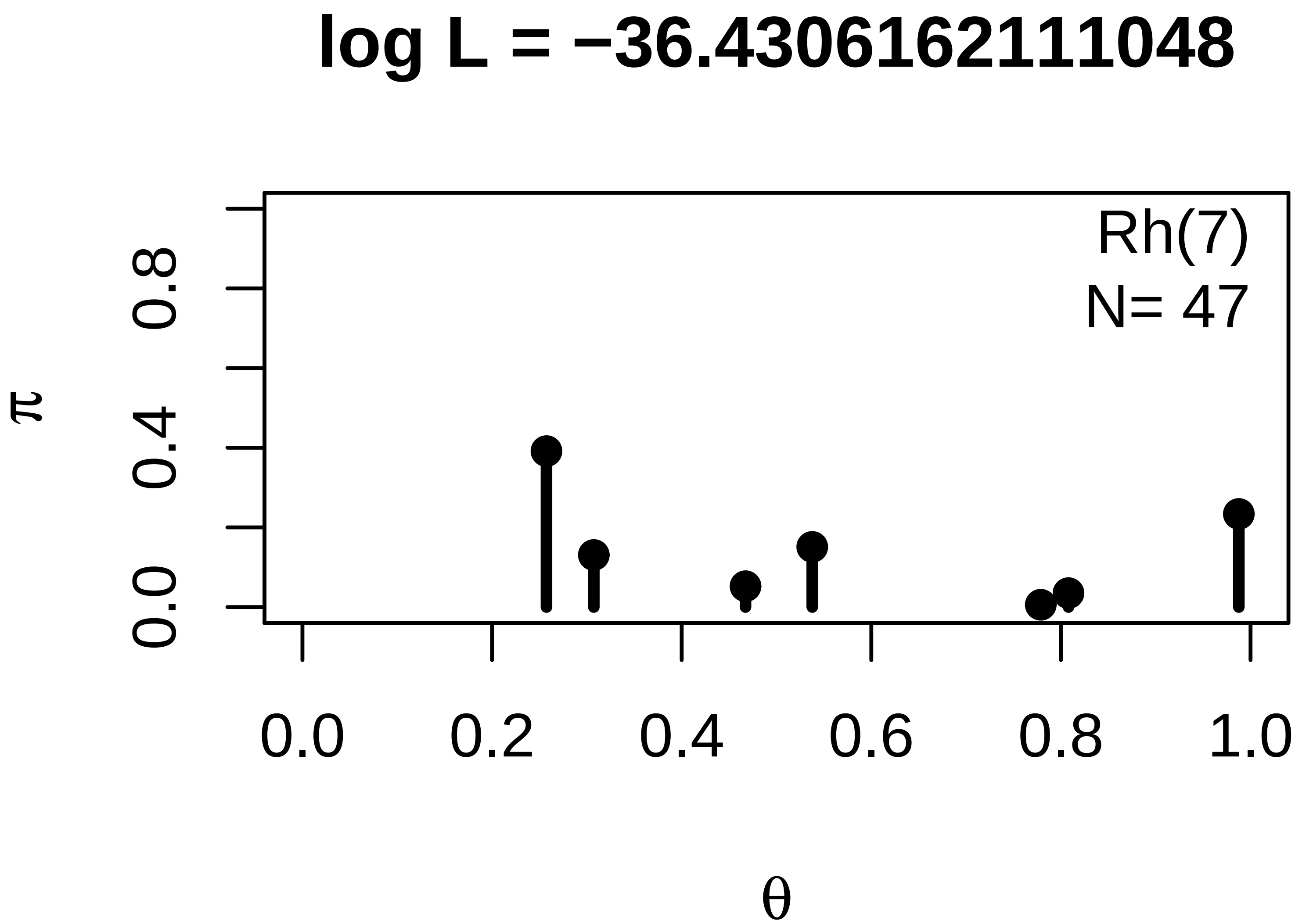
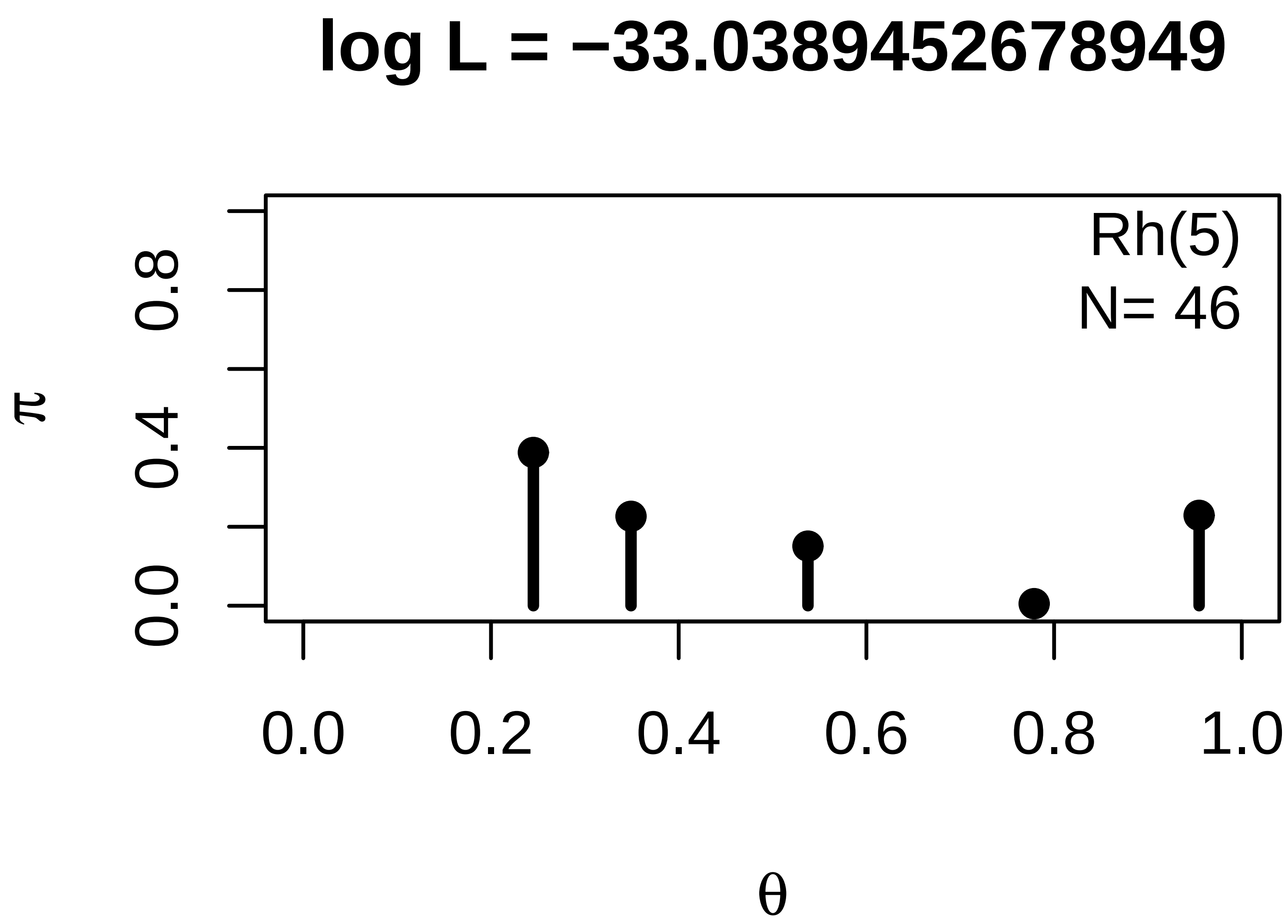


Step 22



Step 23



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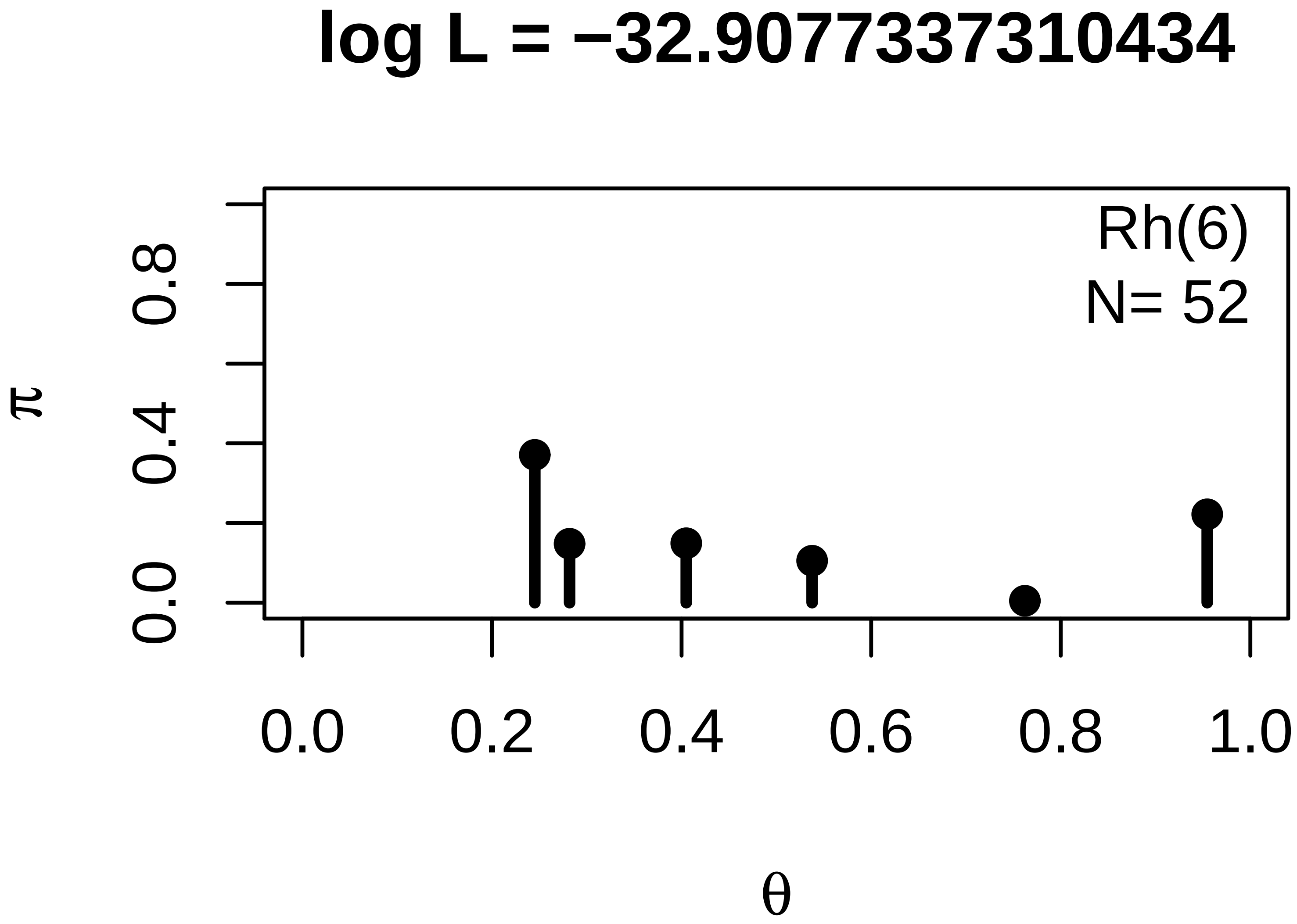

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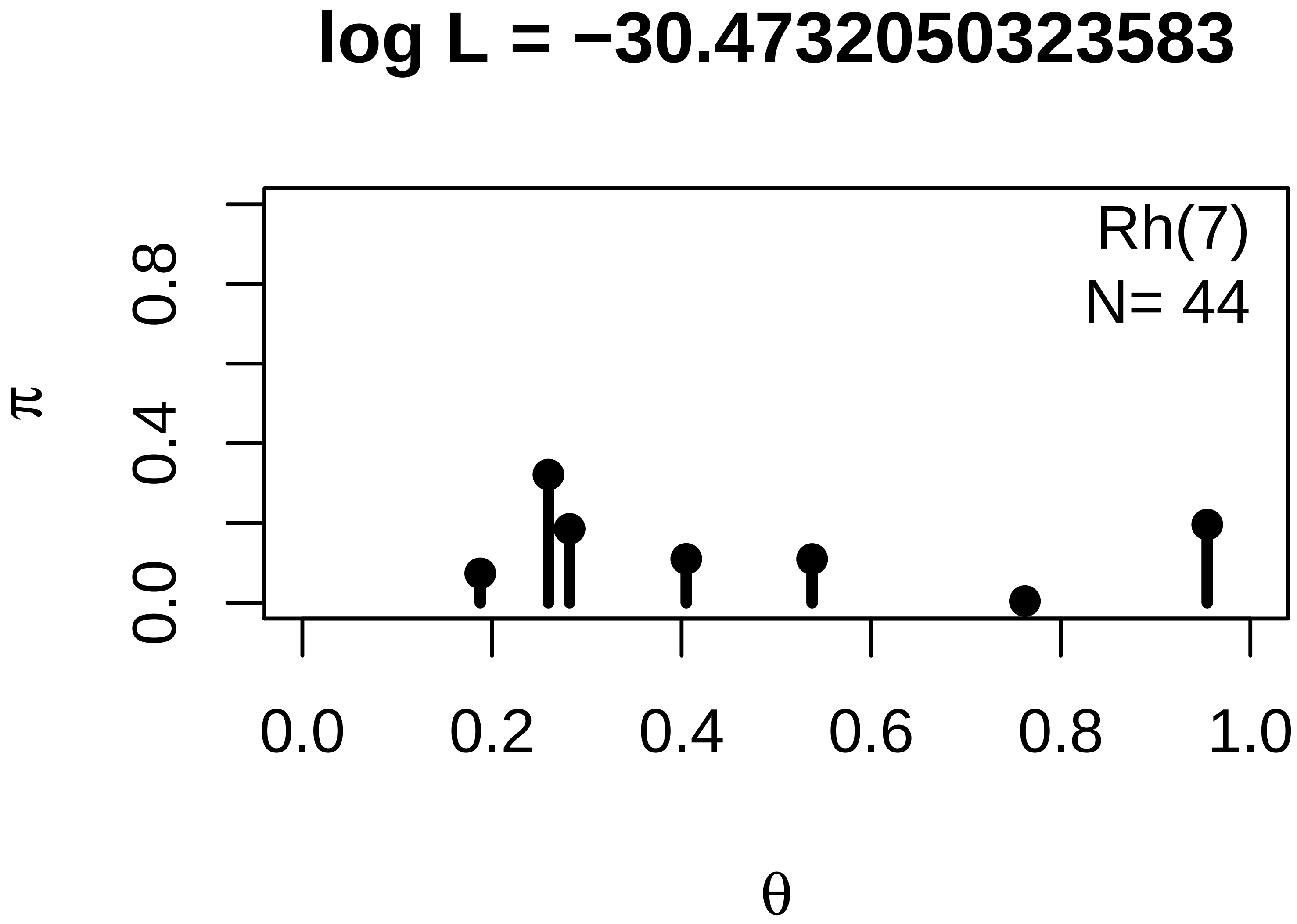

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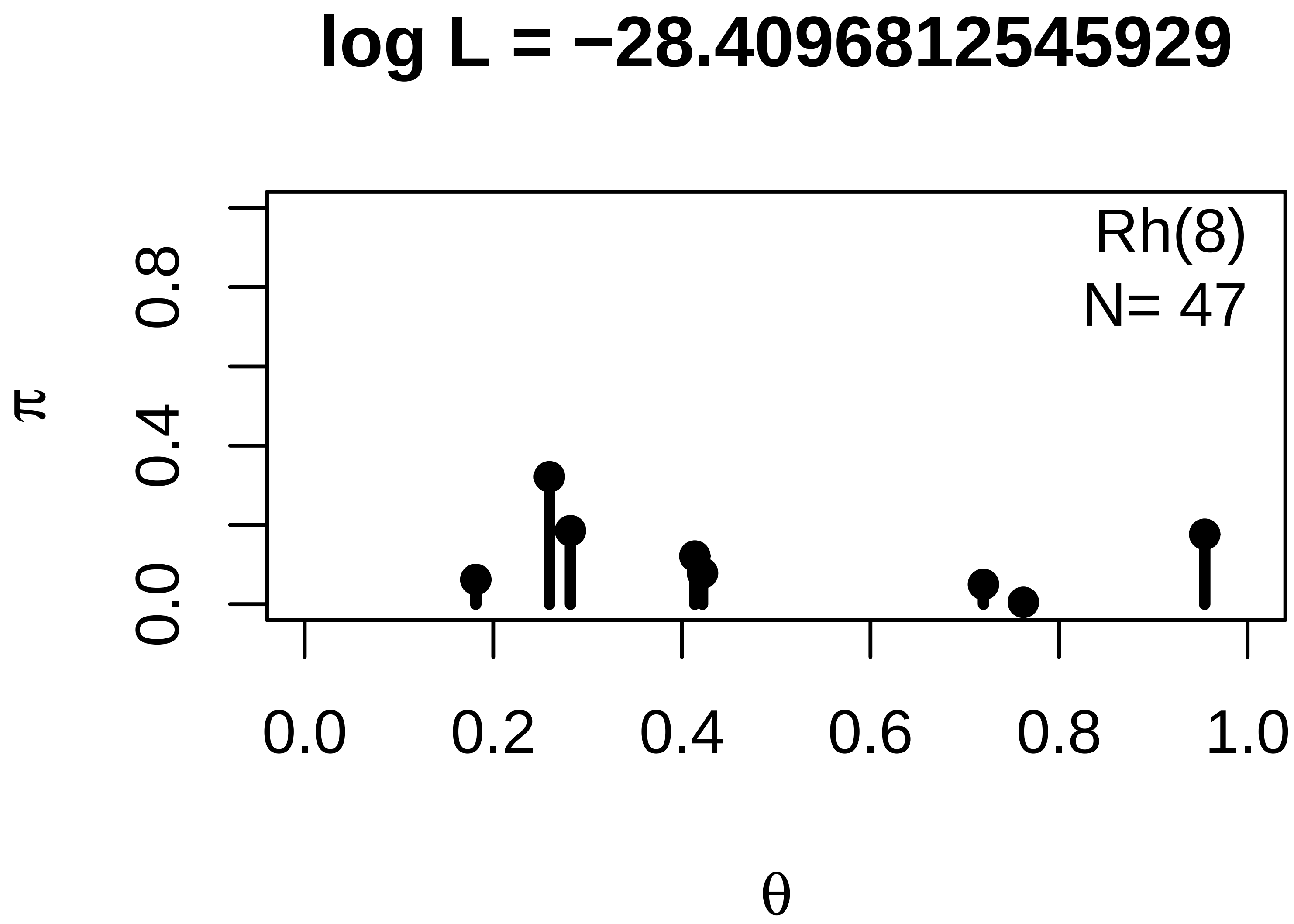

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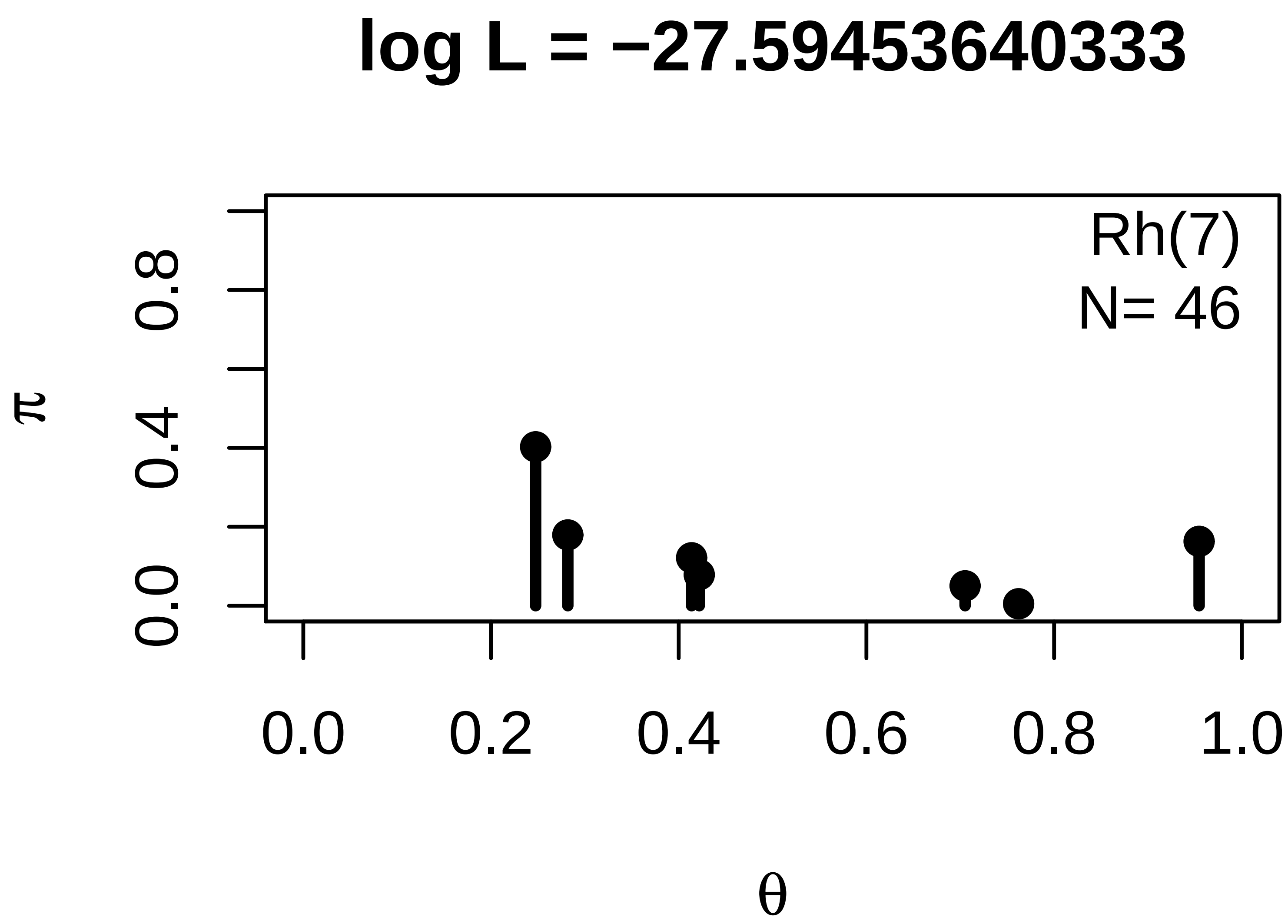

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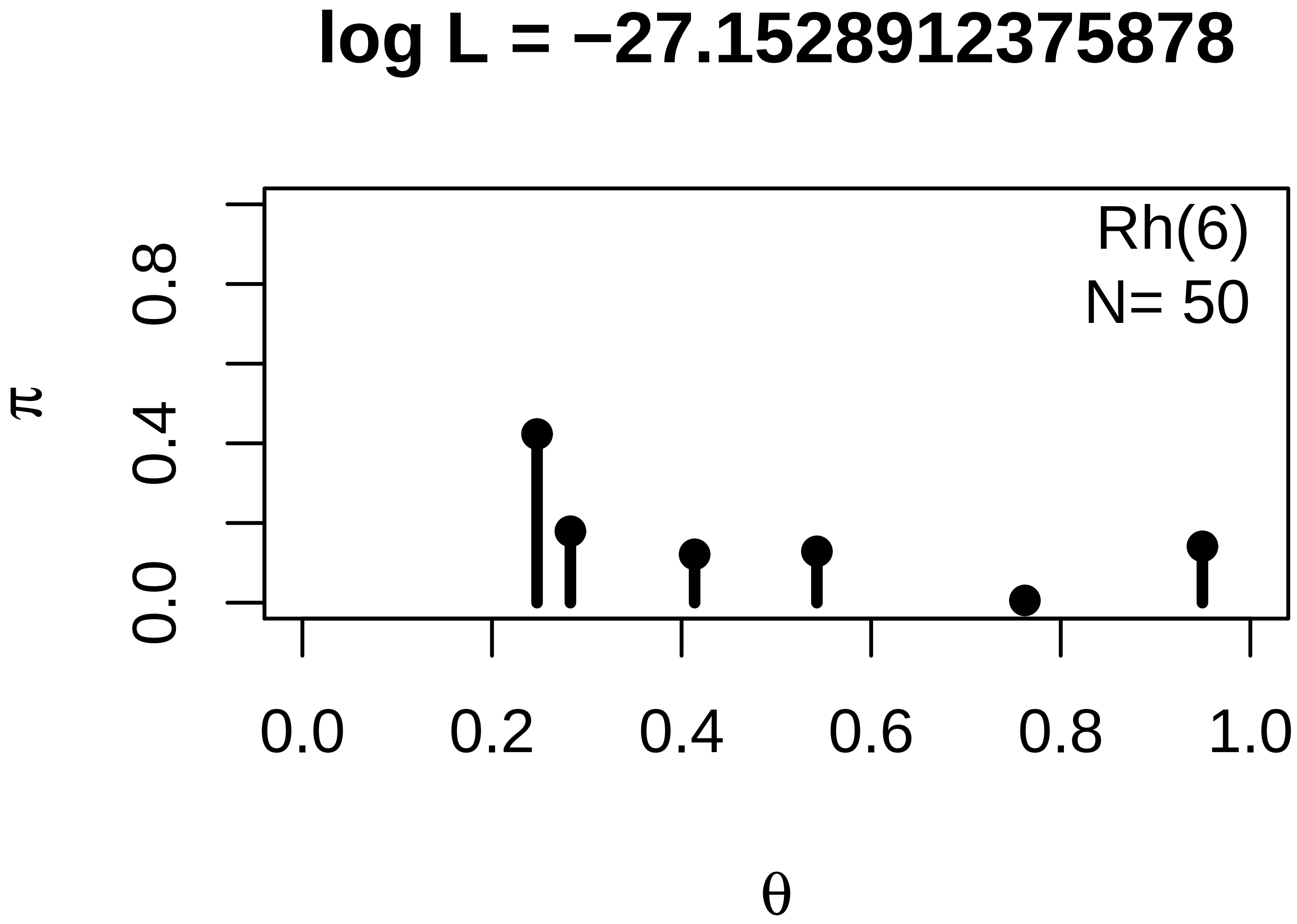

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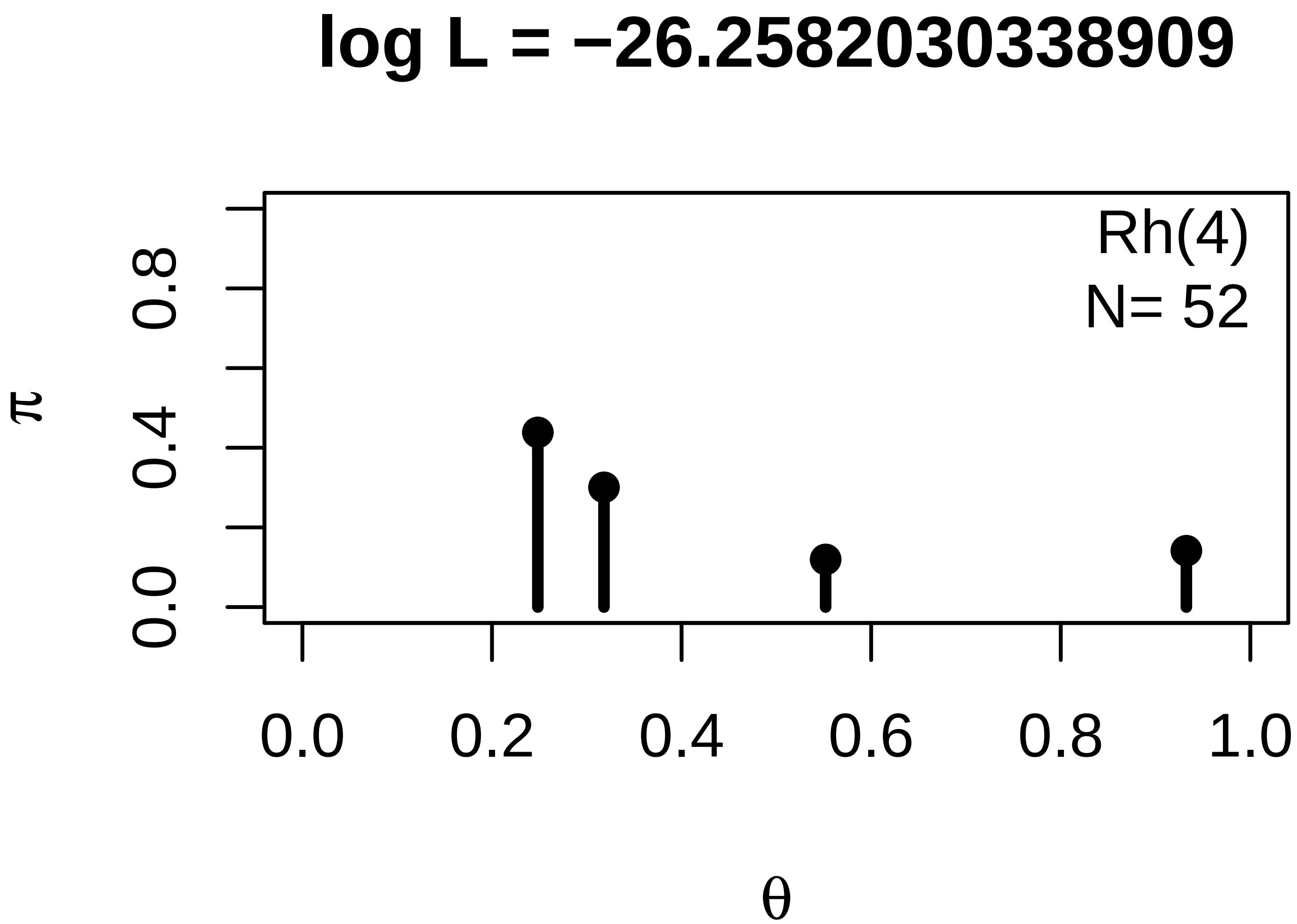

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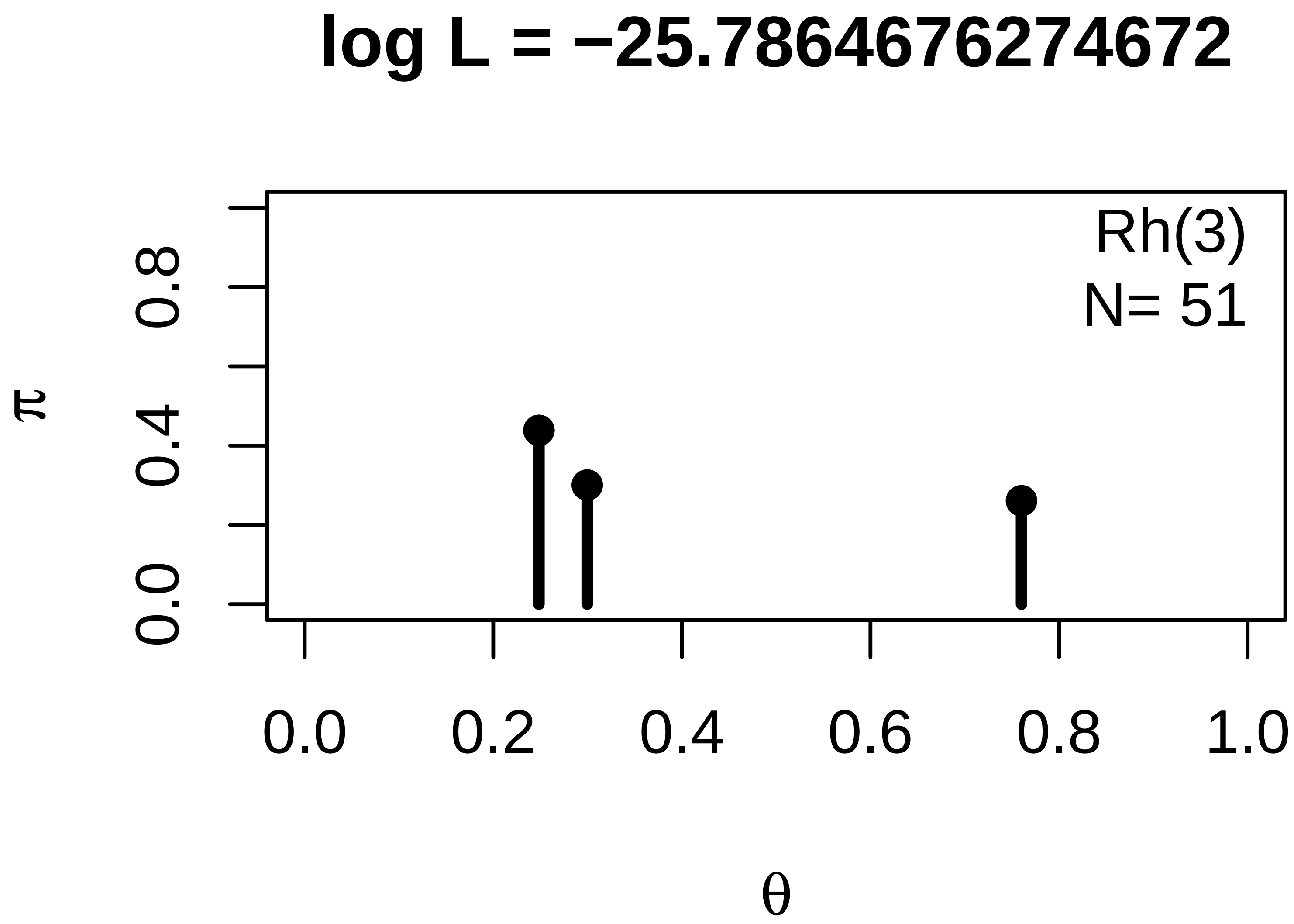

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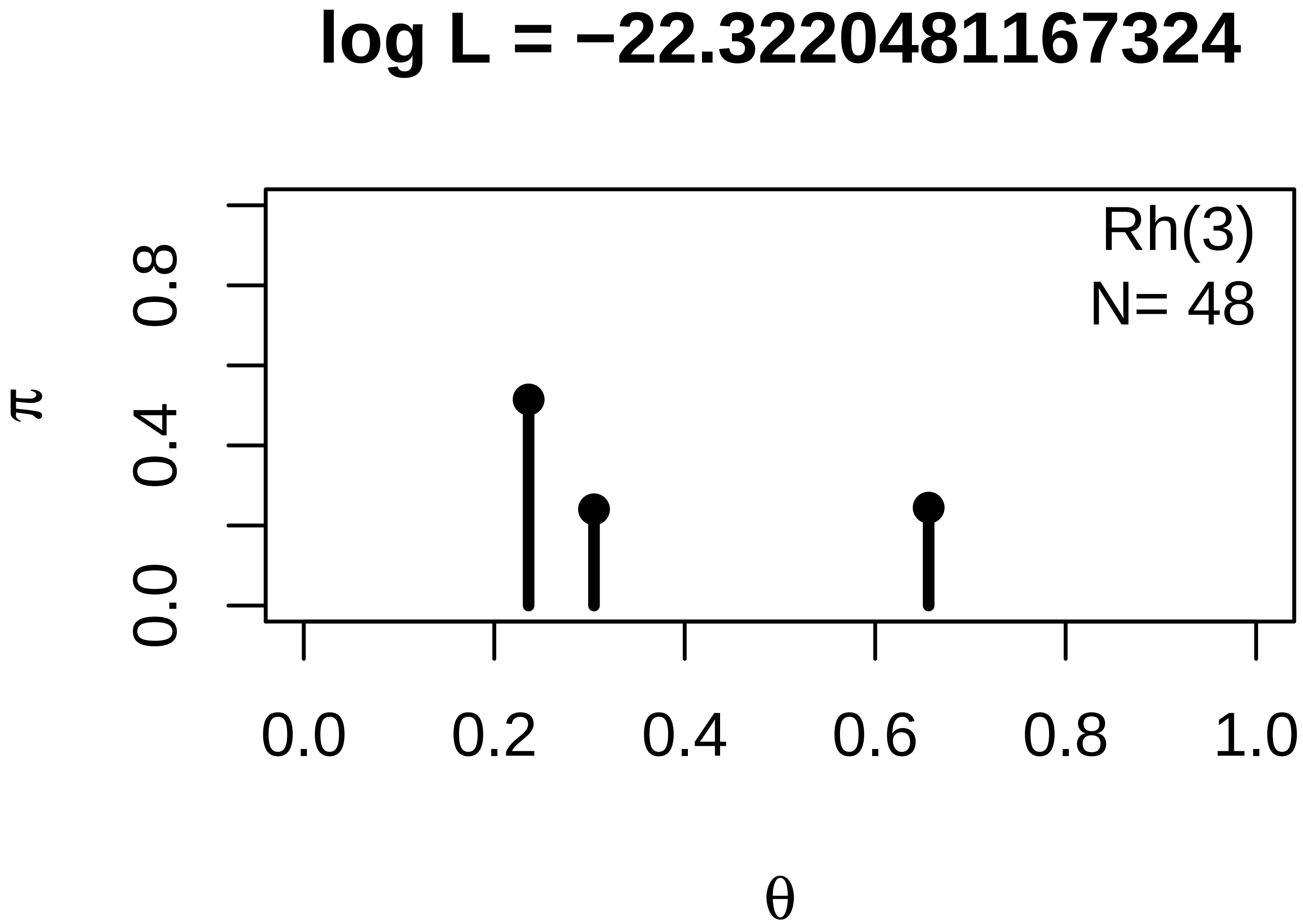

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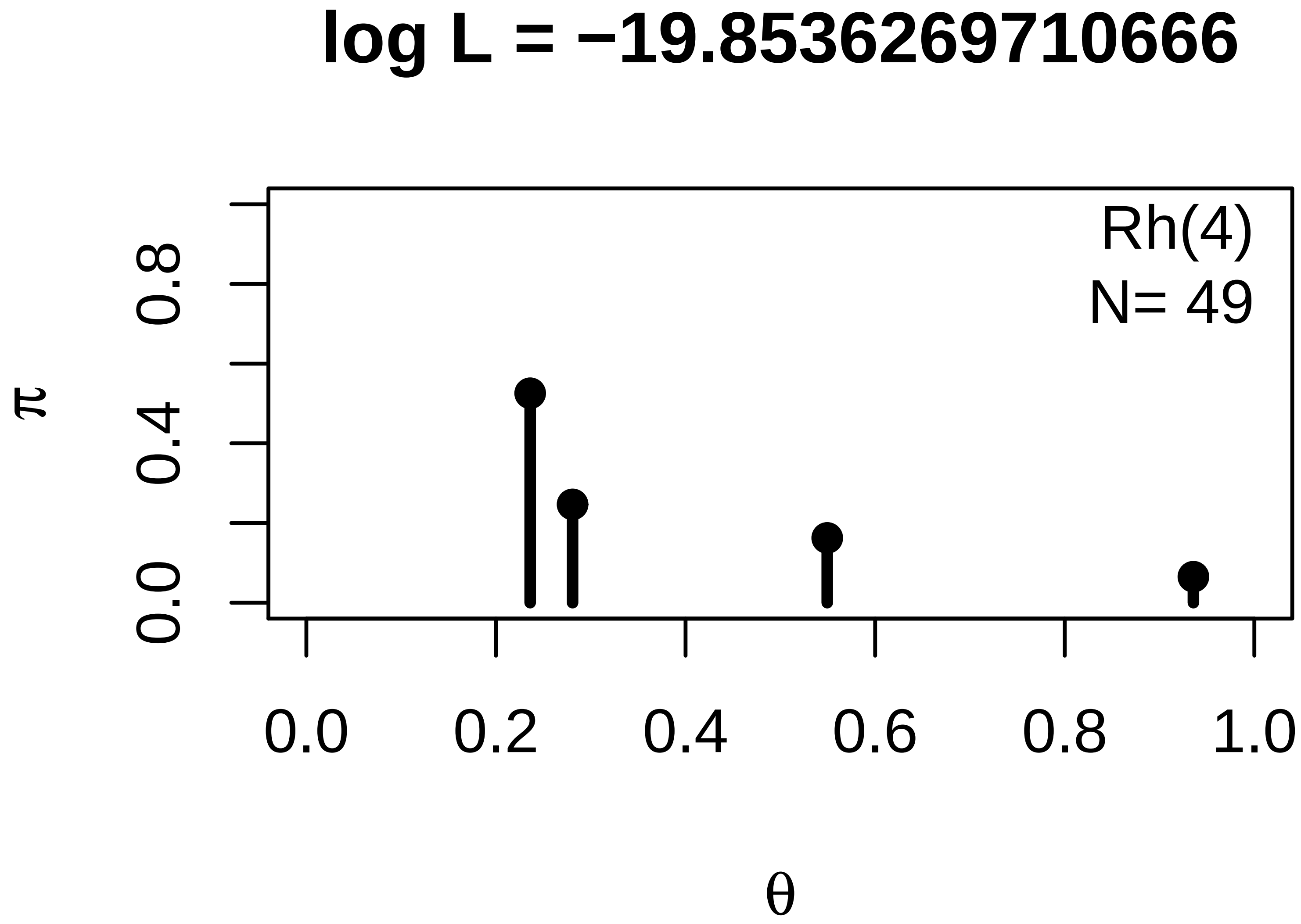

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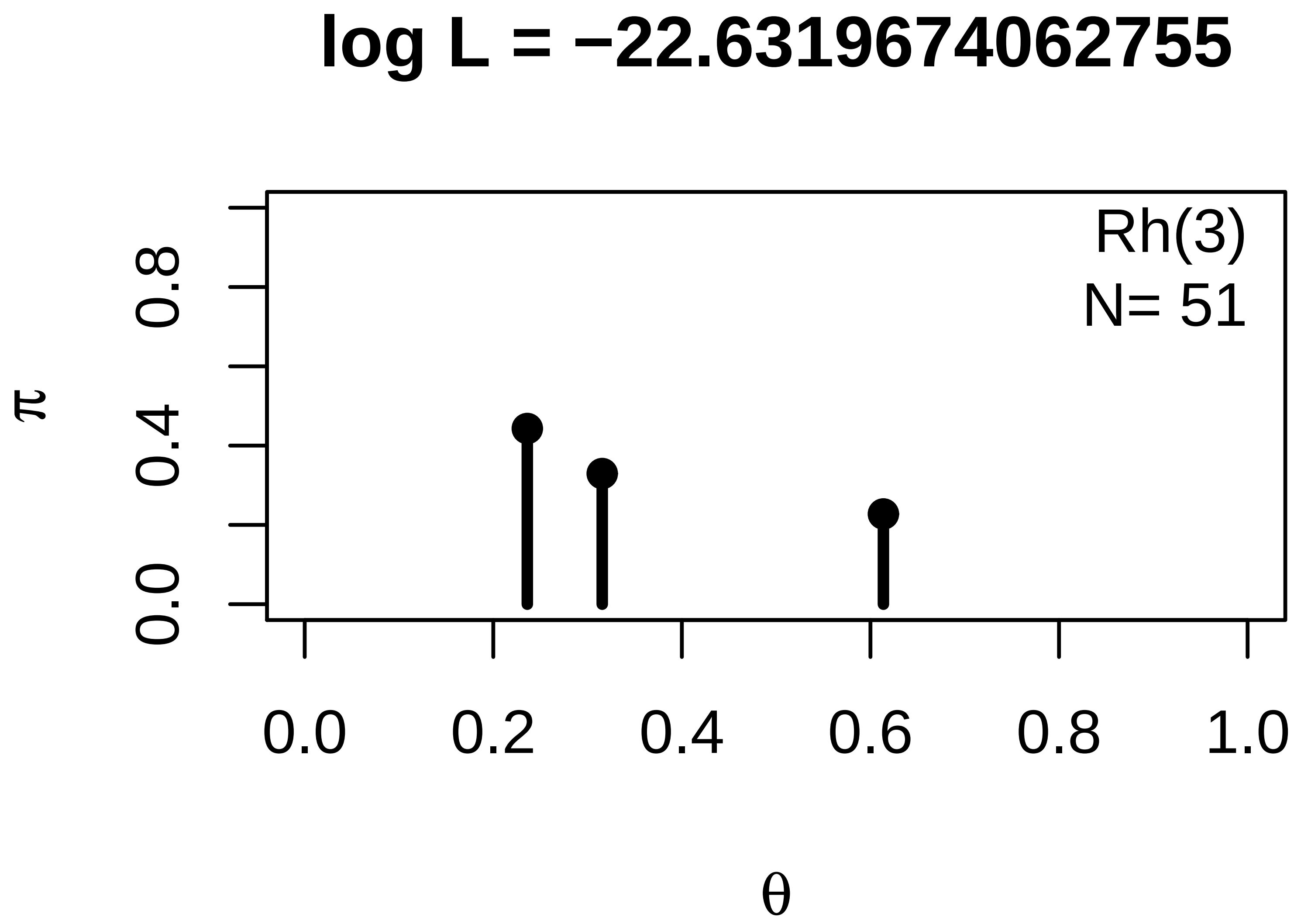

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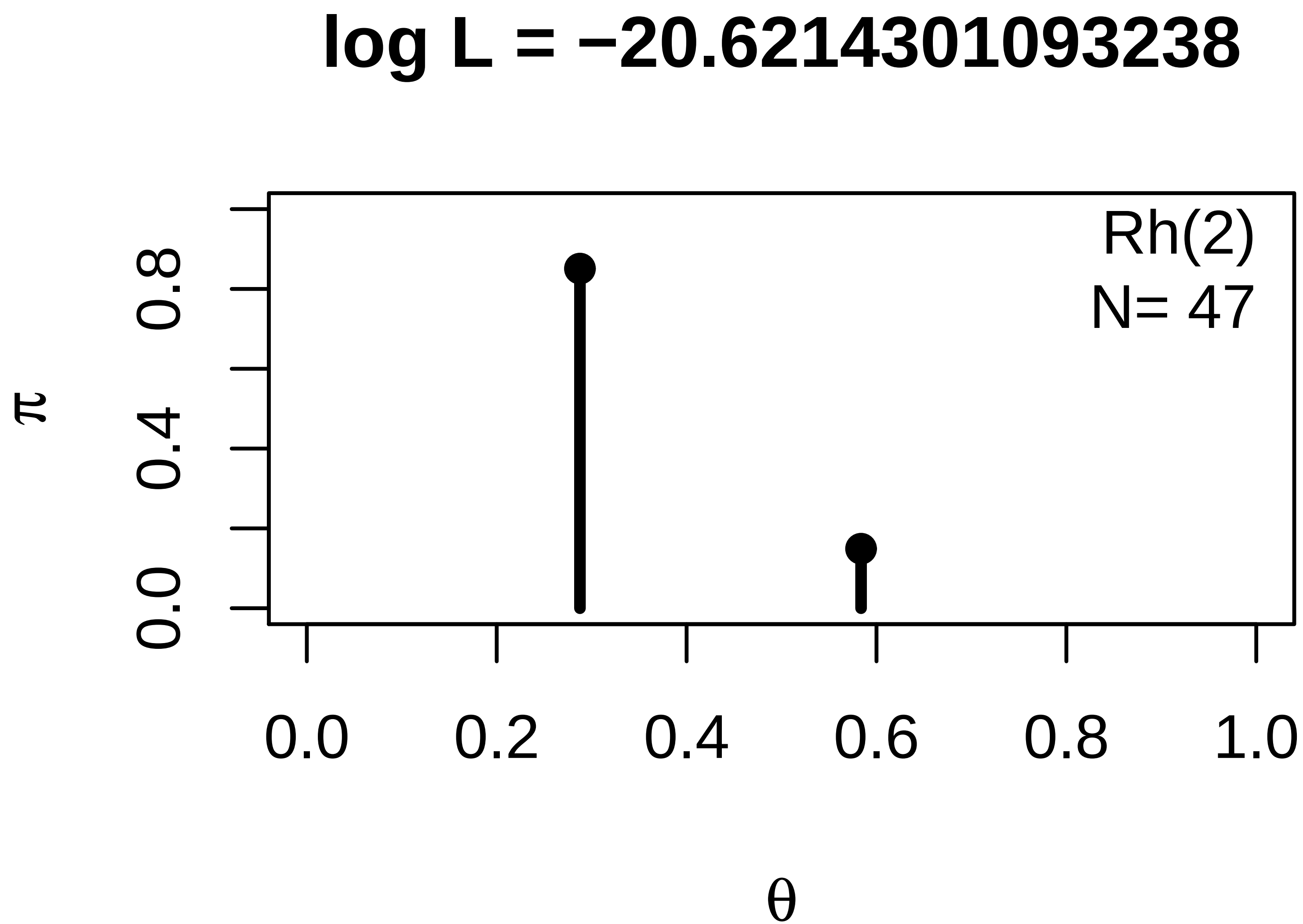

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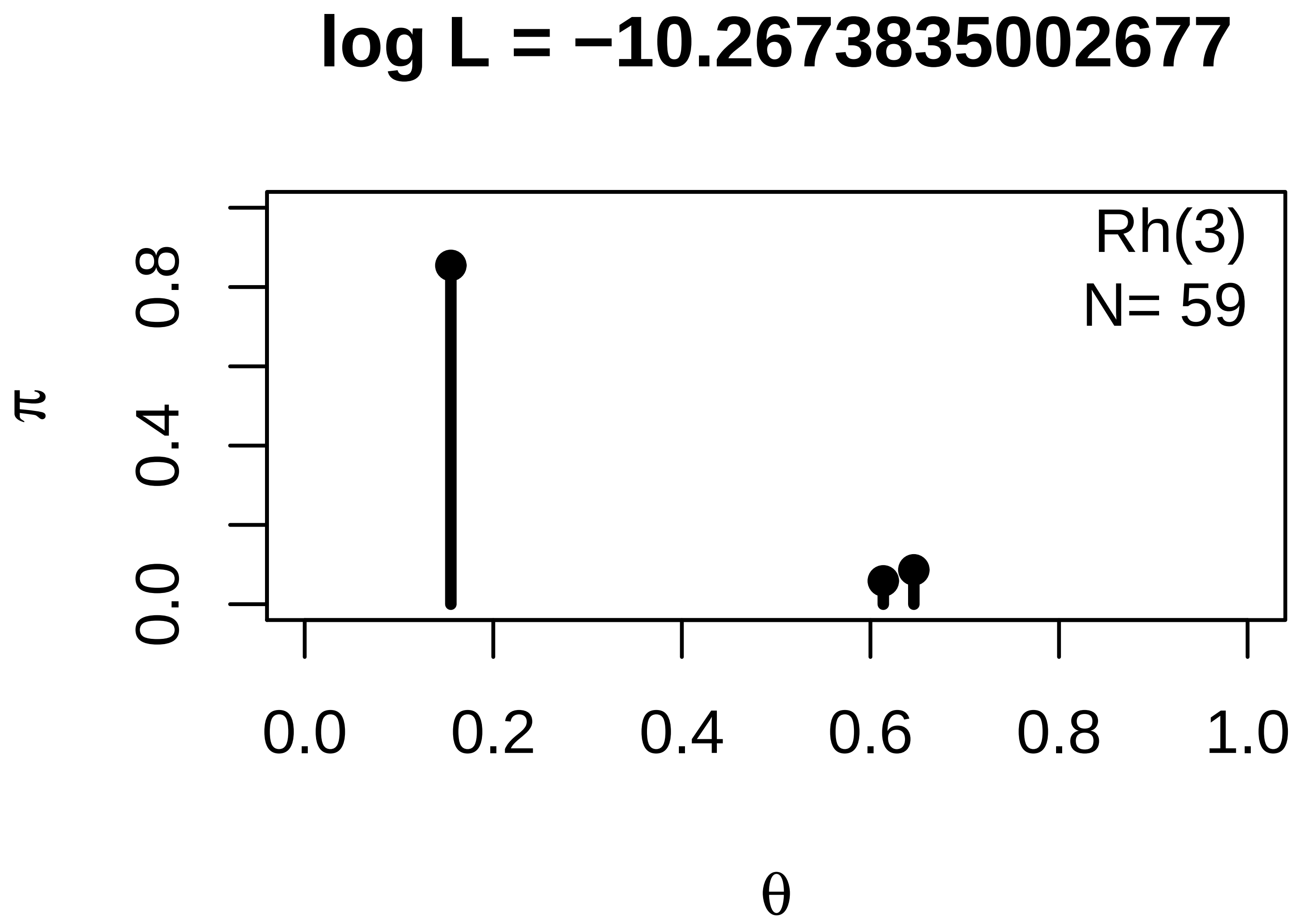

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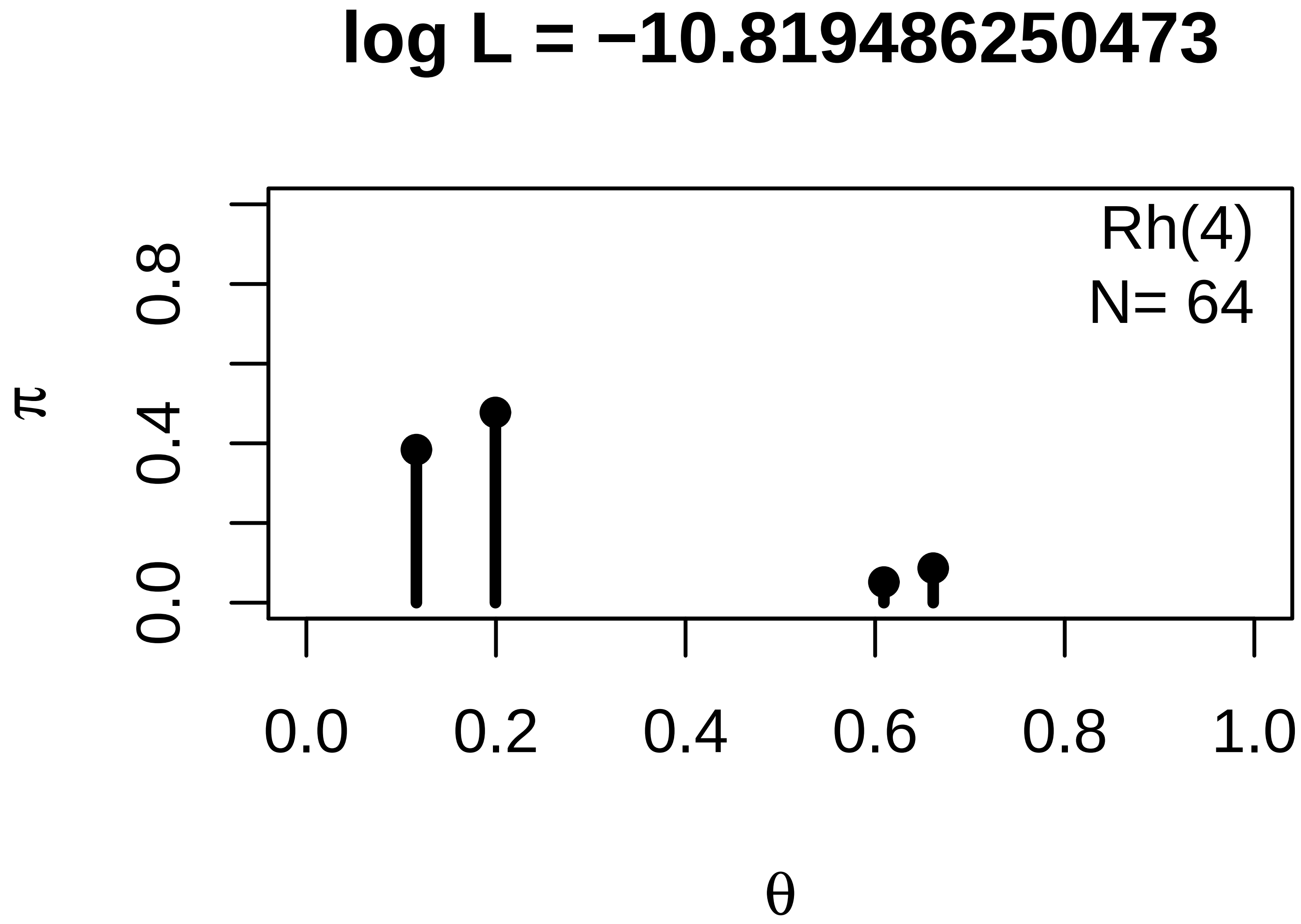

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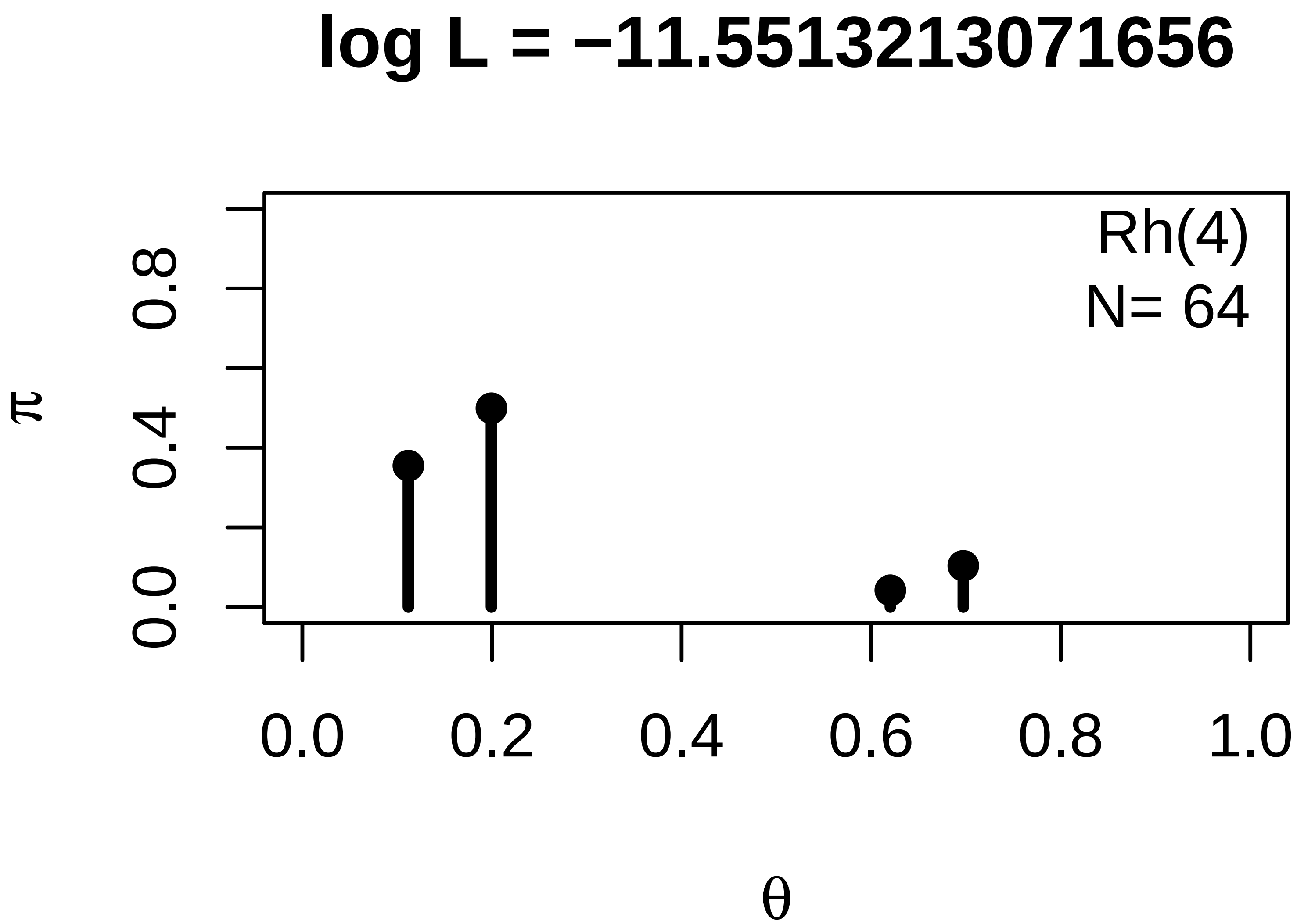

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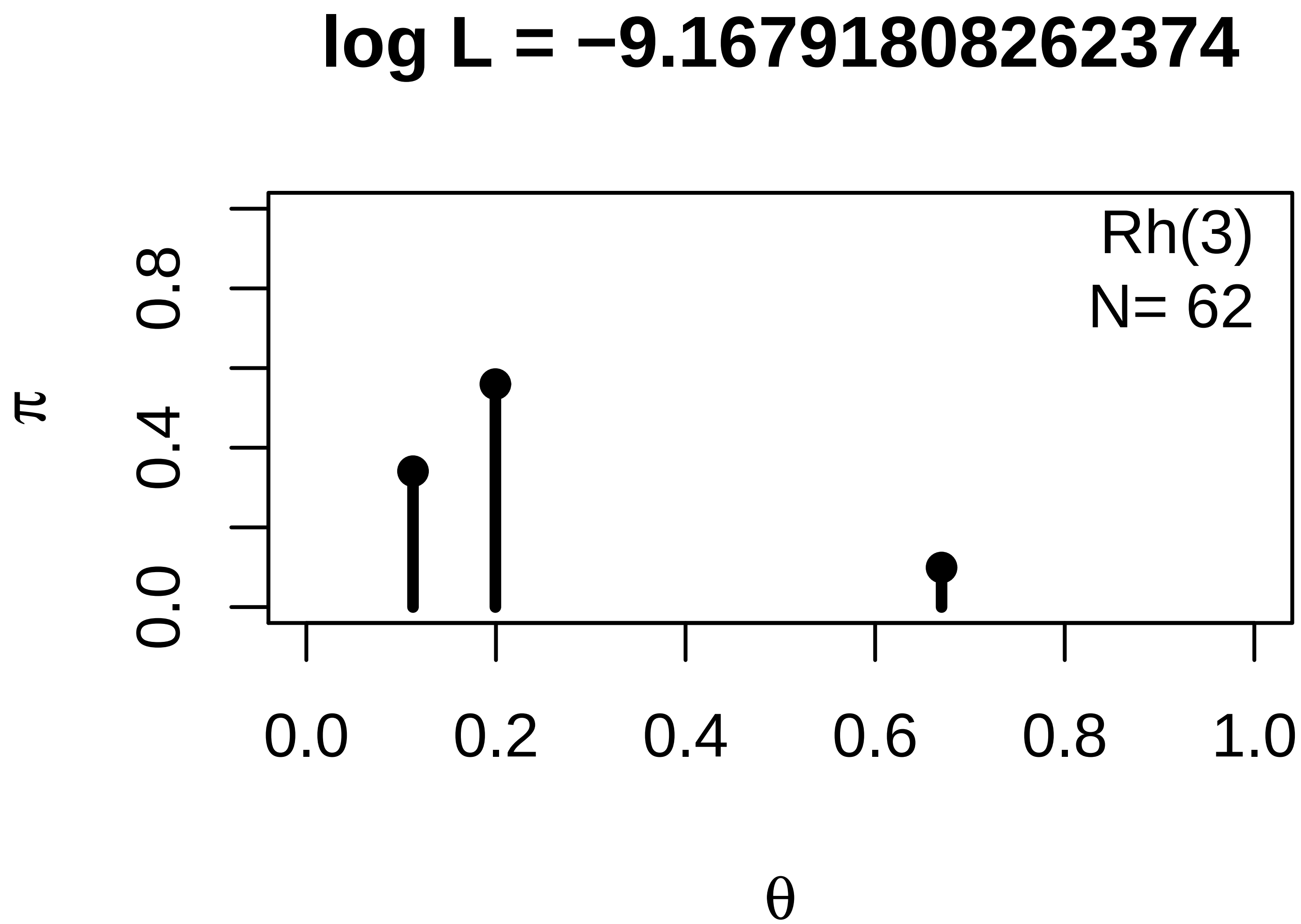

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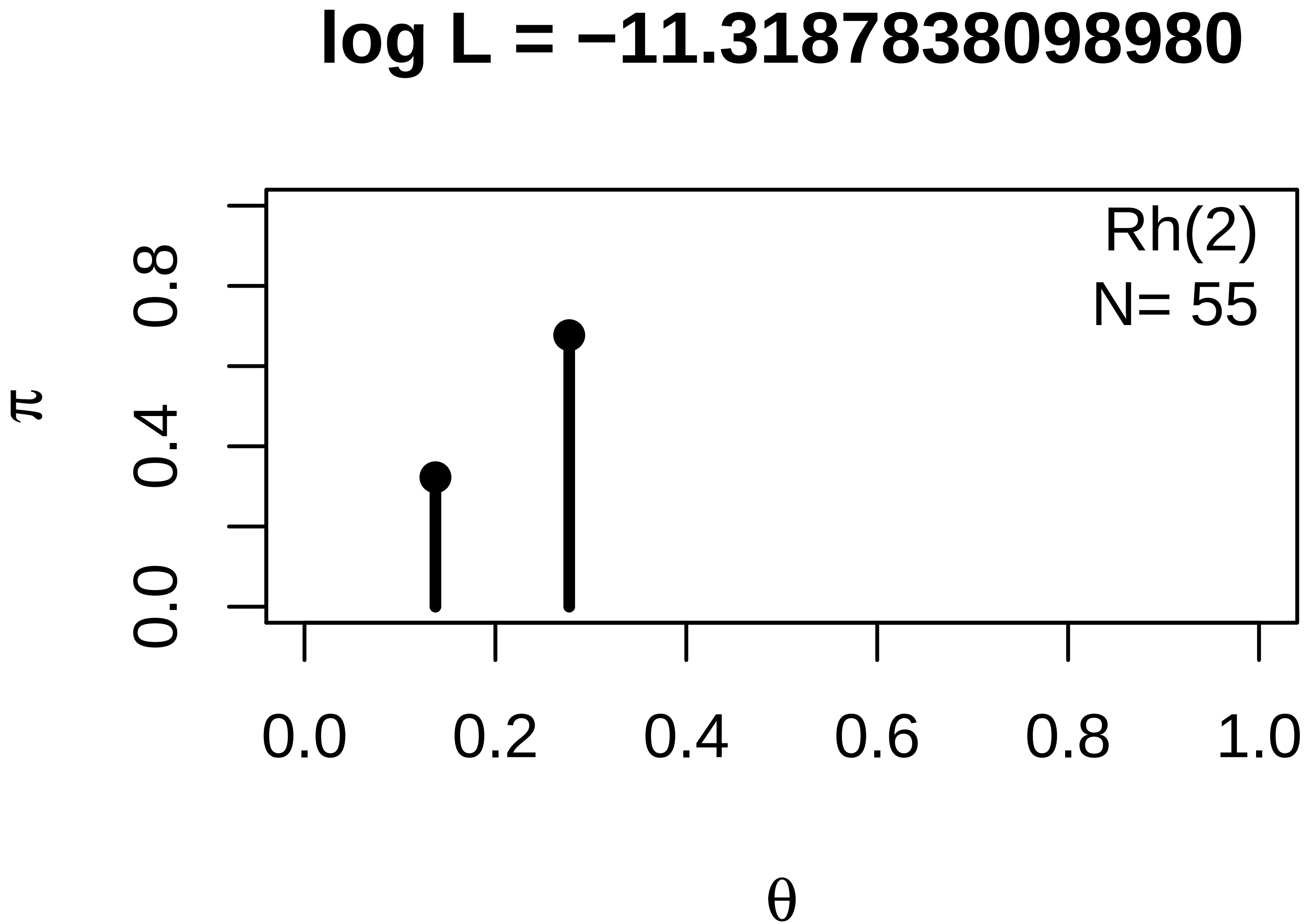

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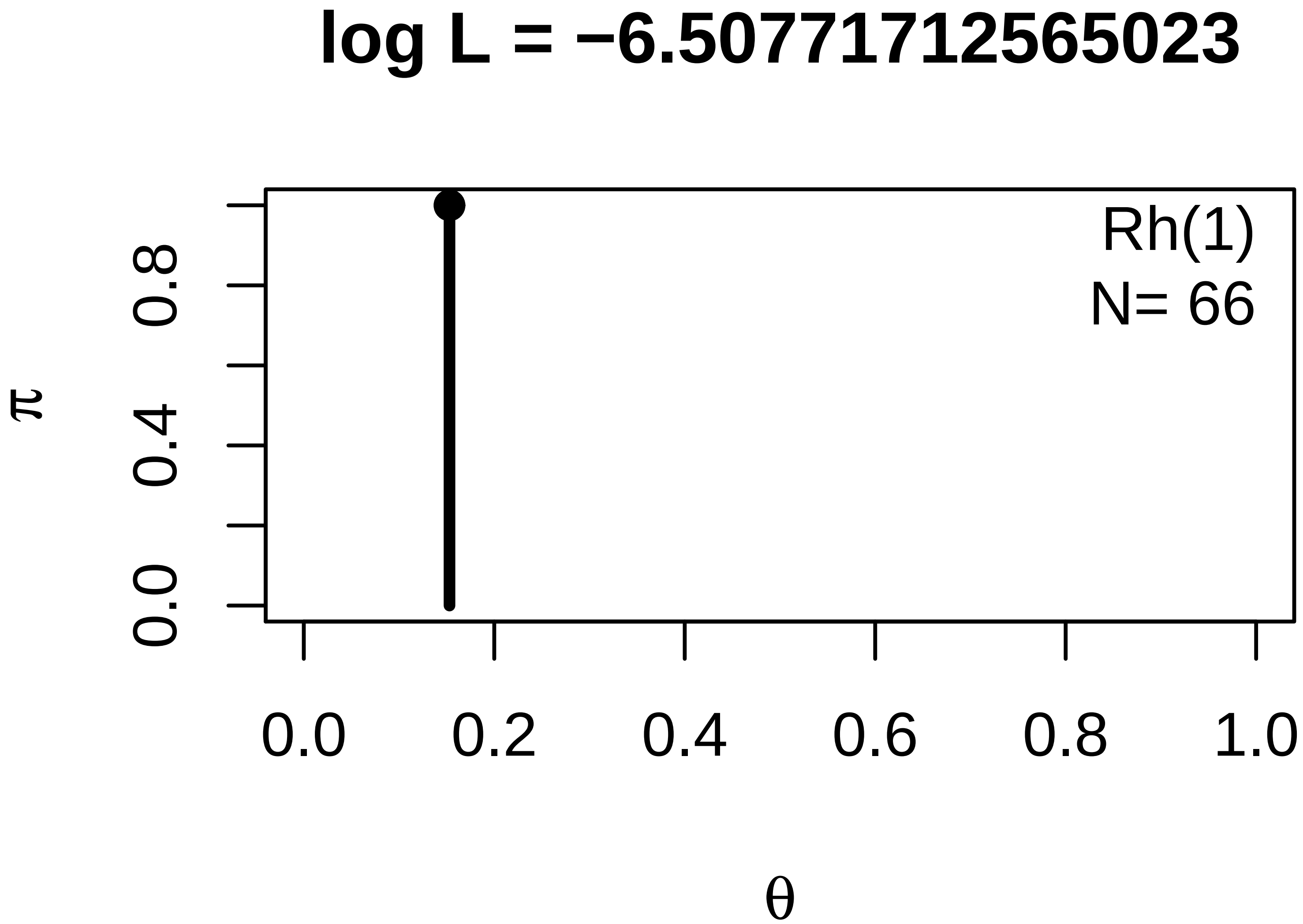

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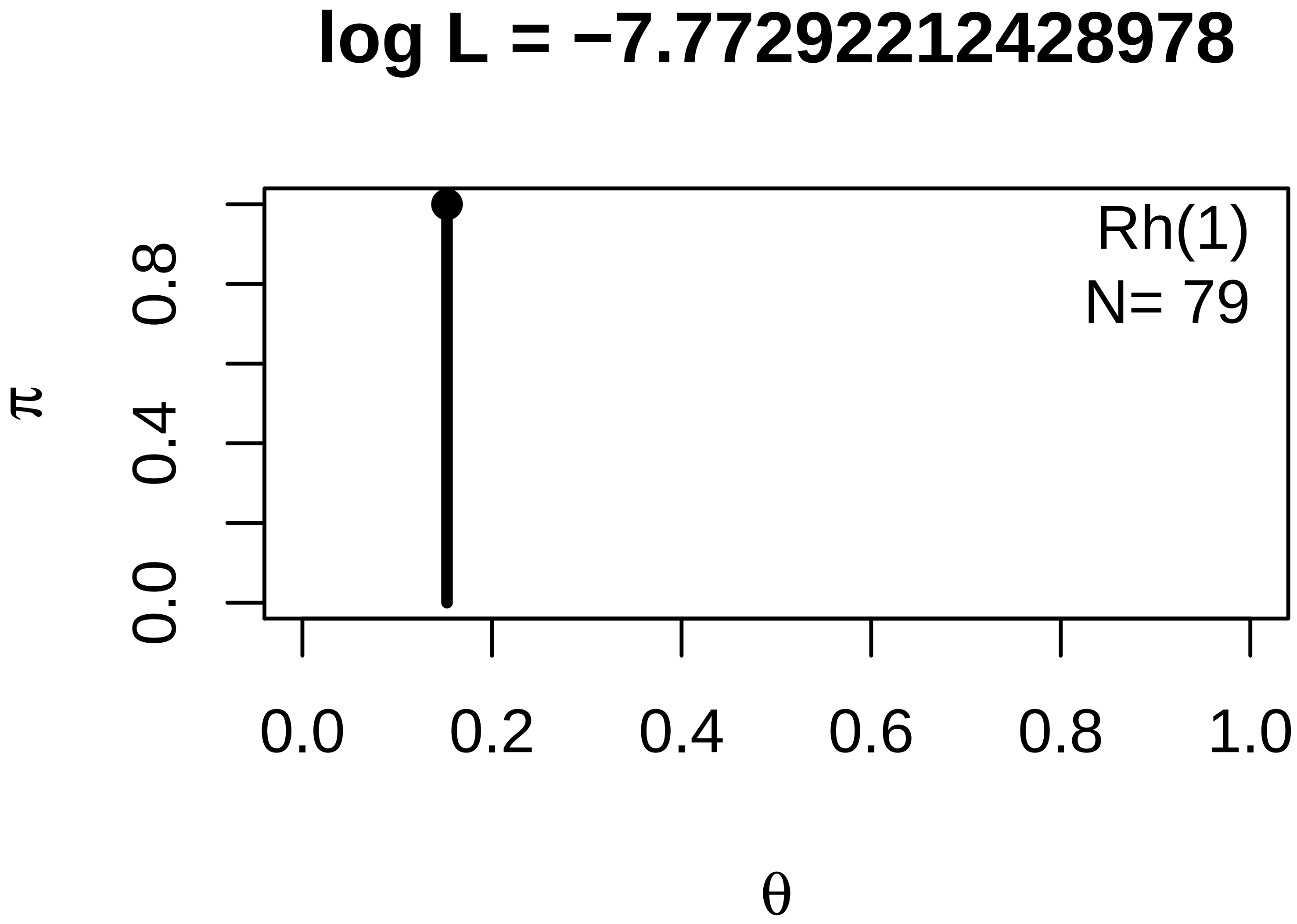

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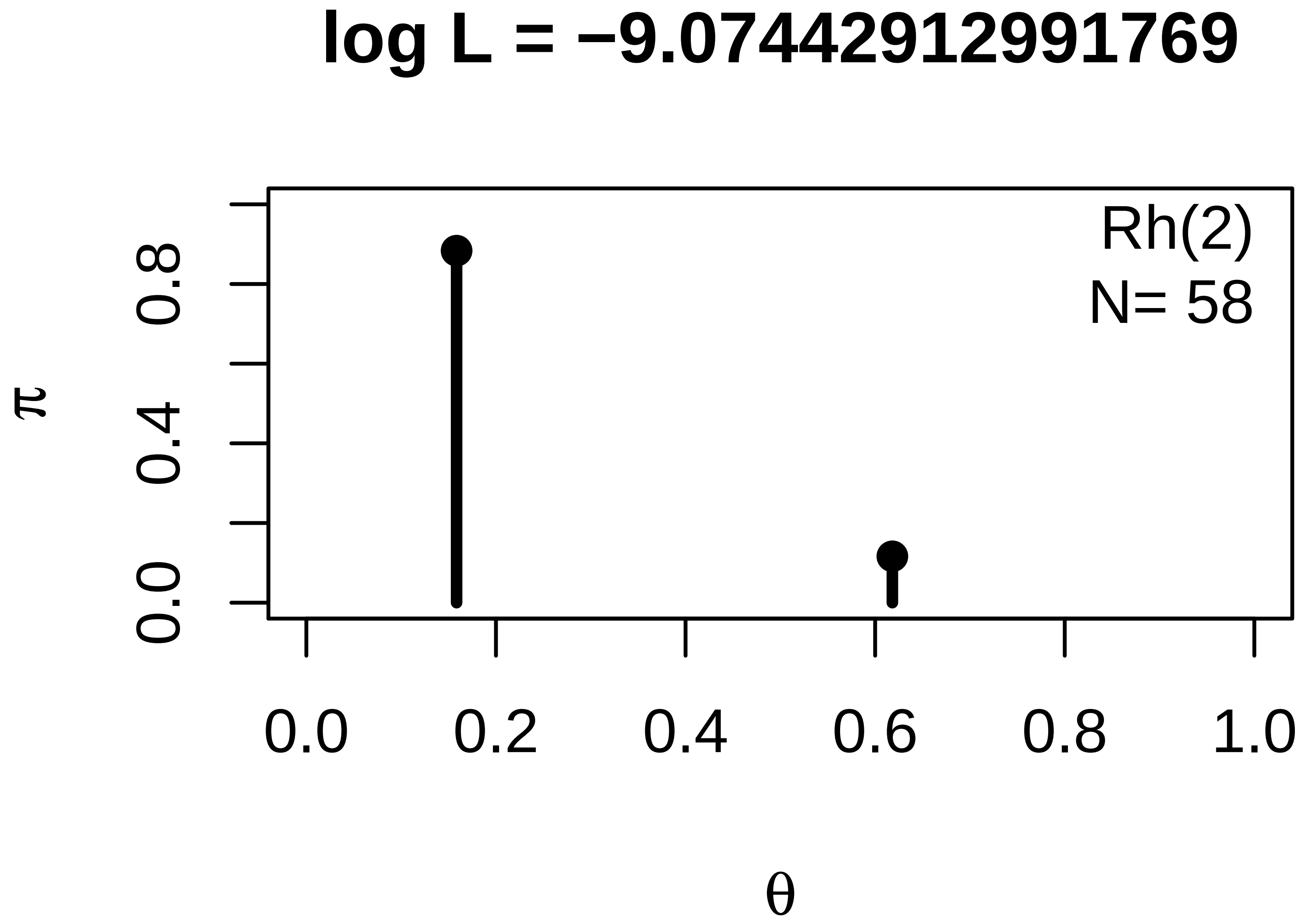

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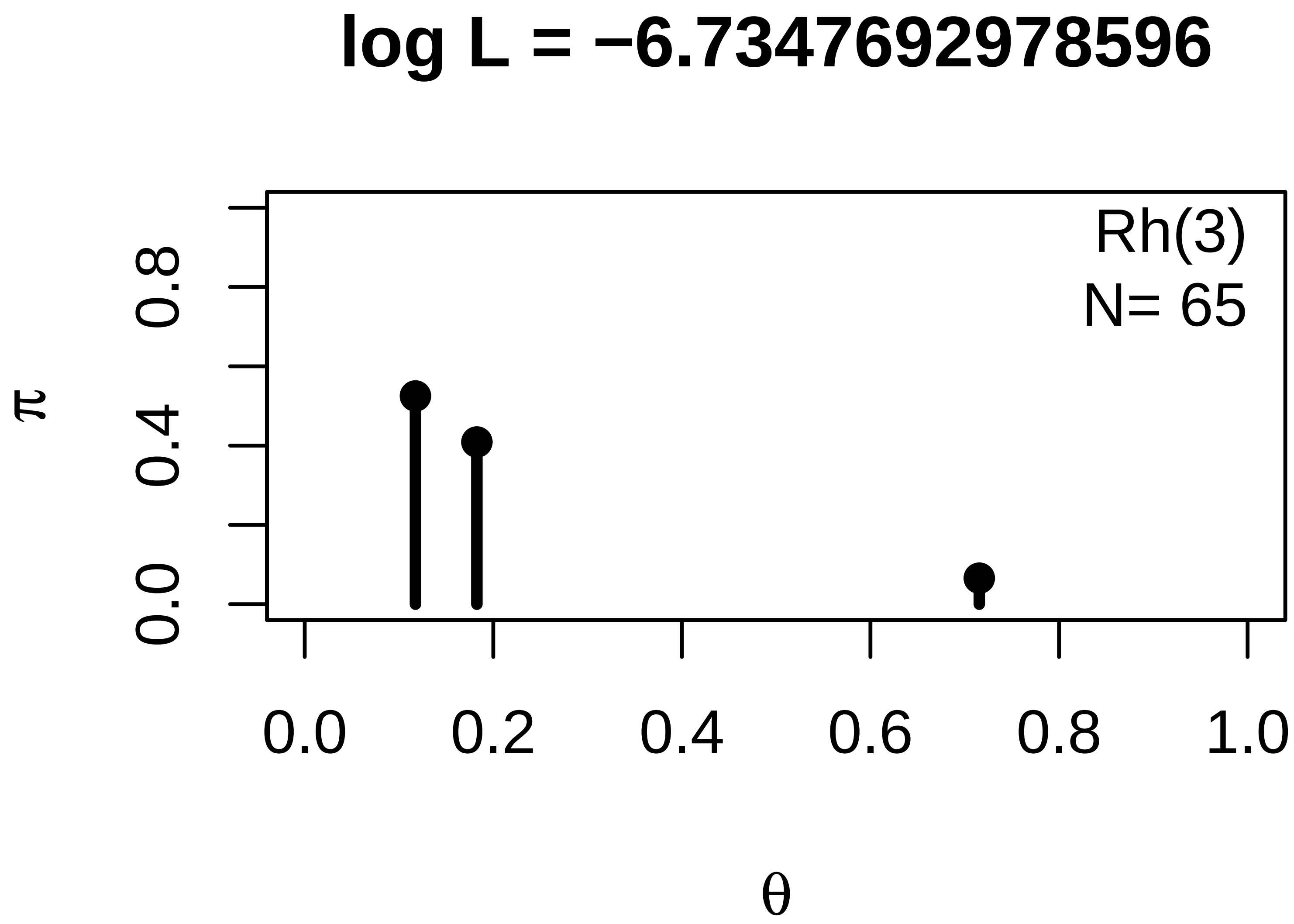

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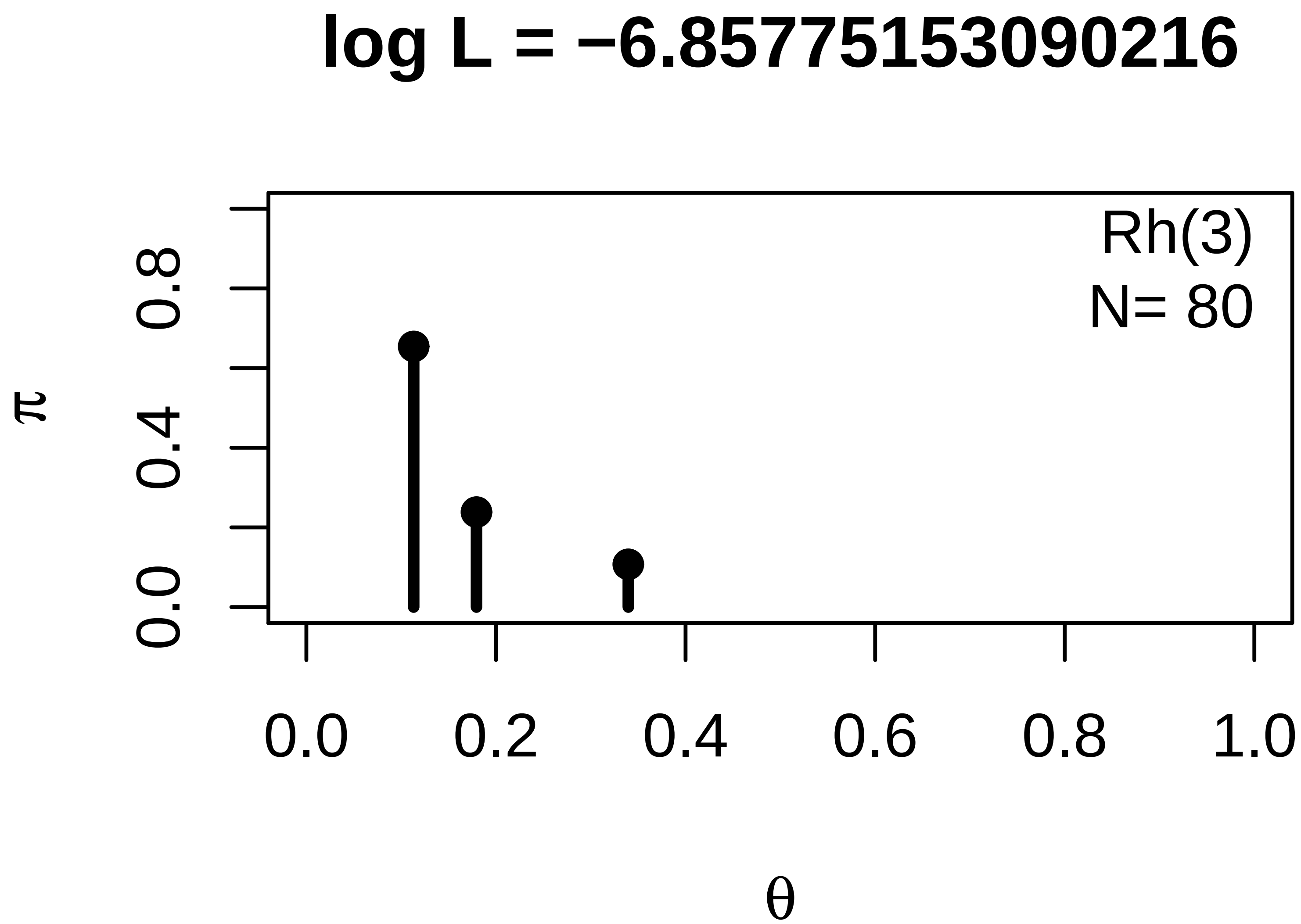

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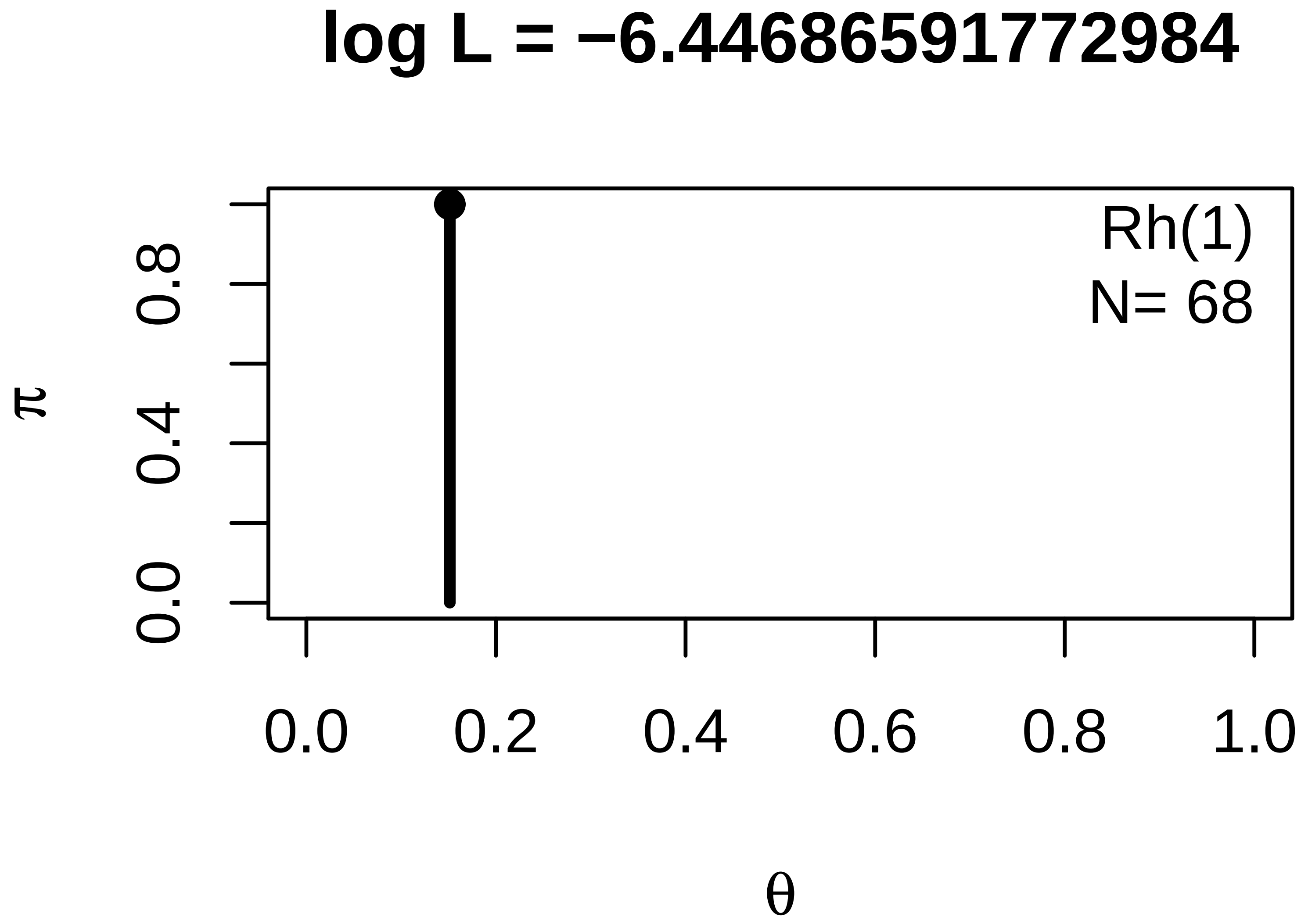

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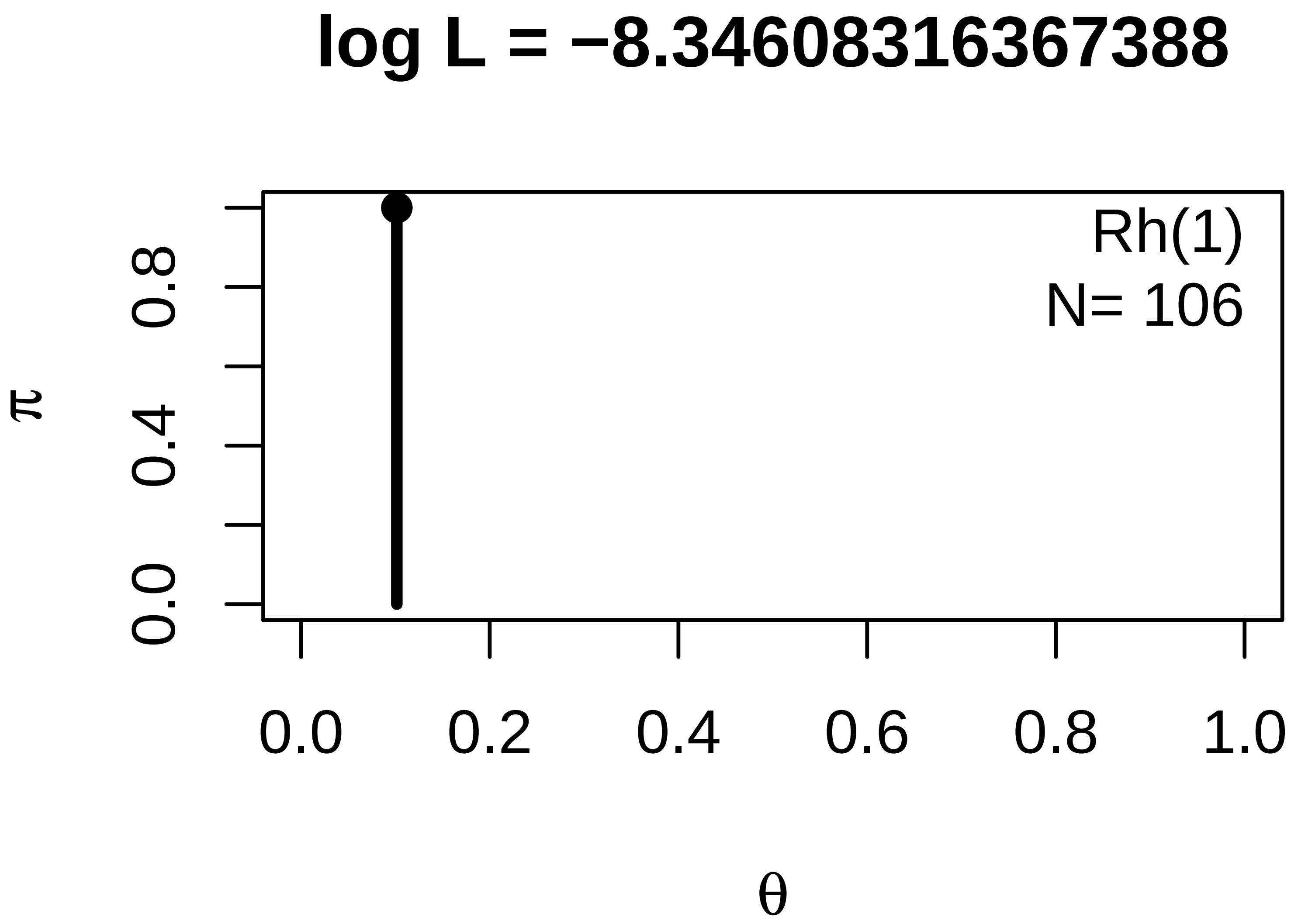

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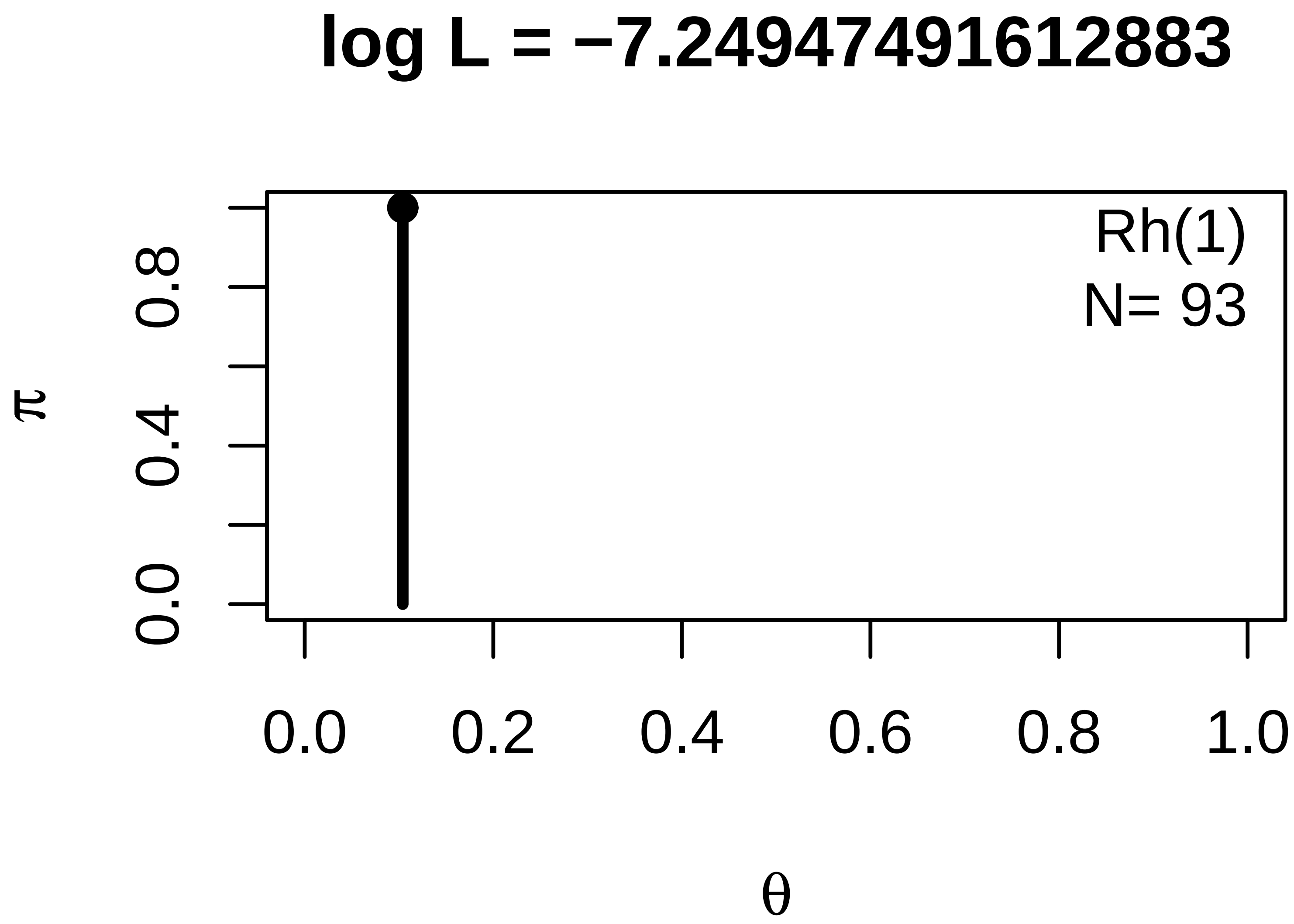

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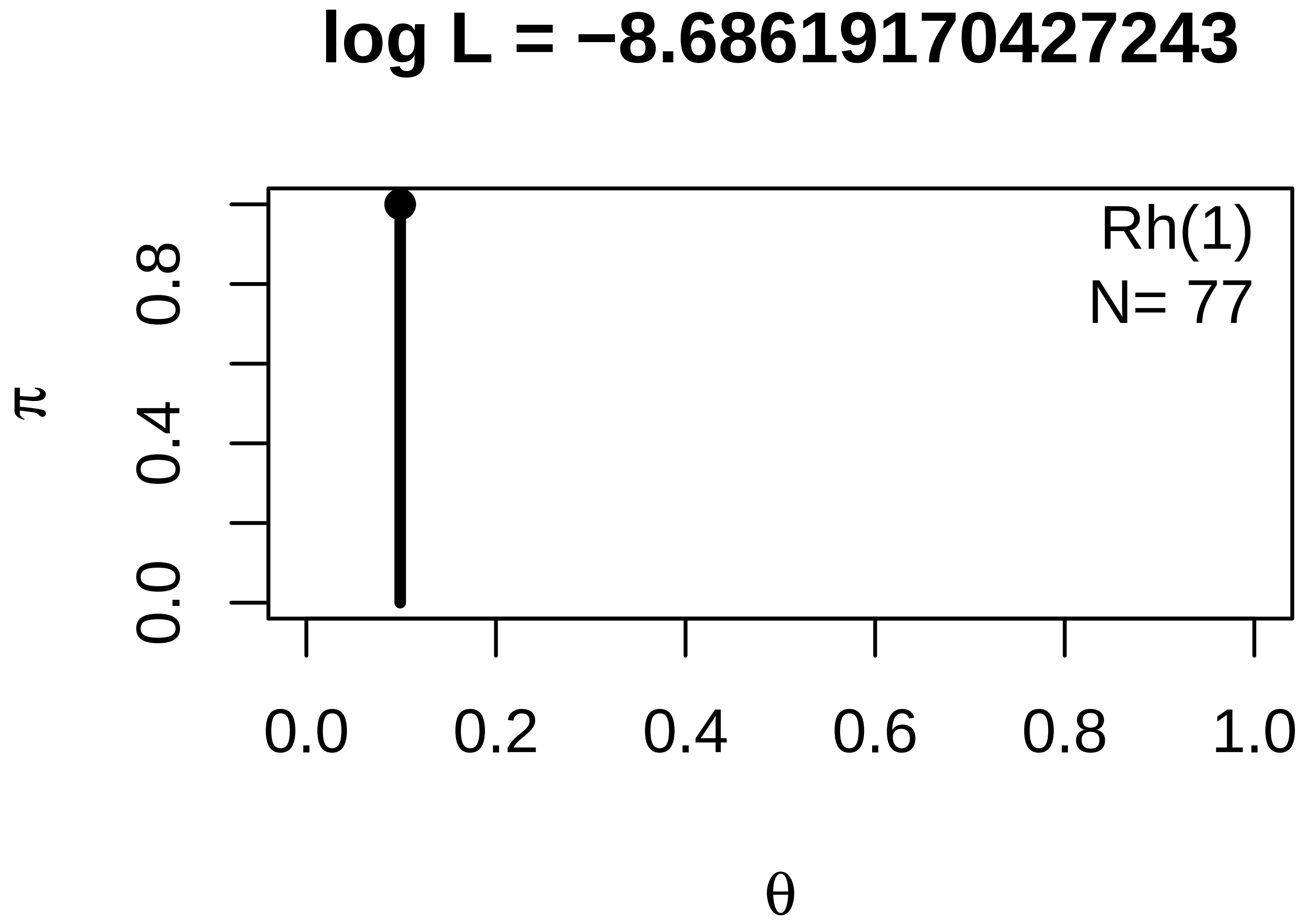

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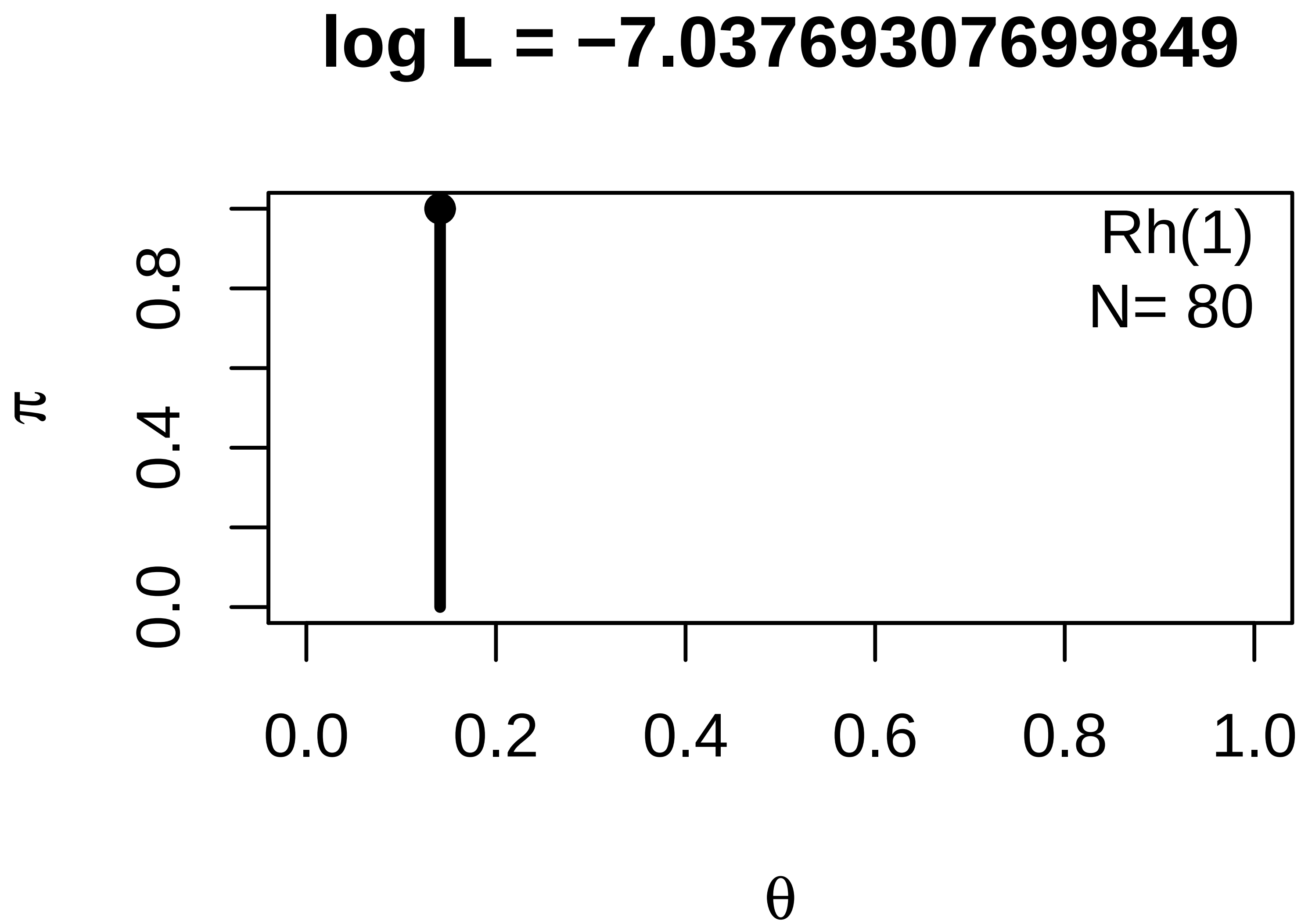

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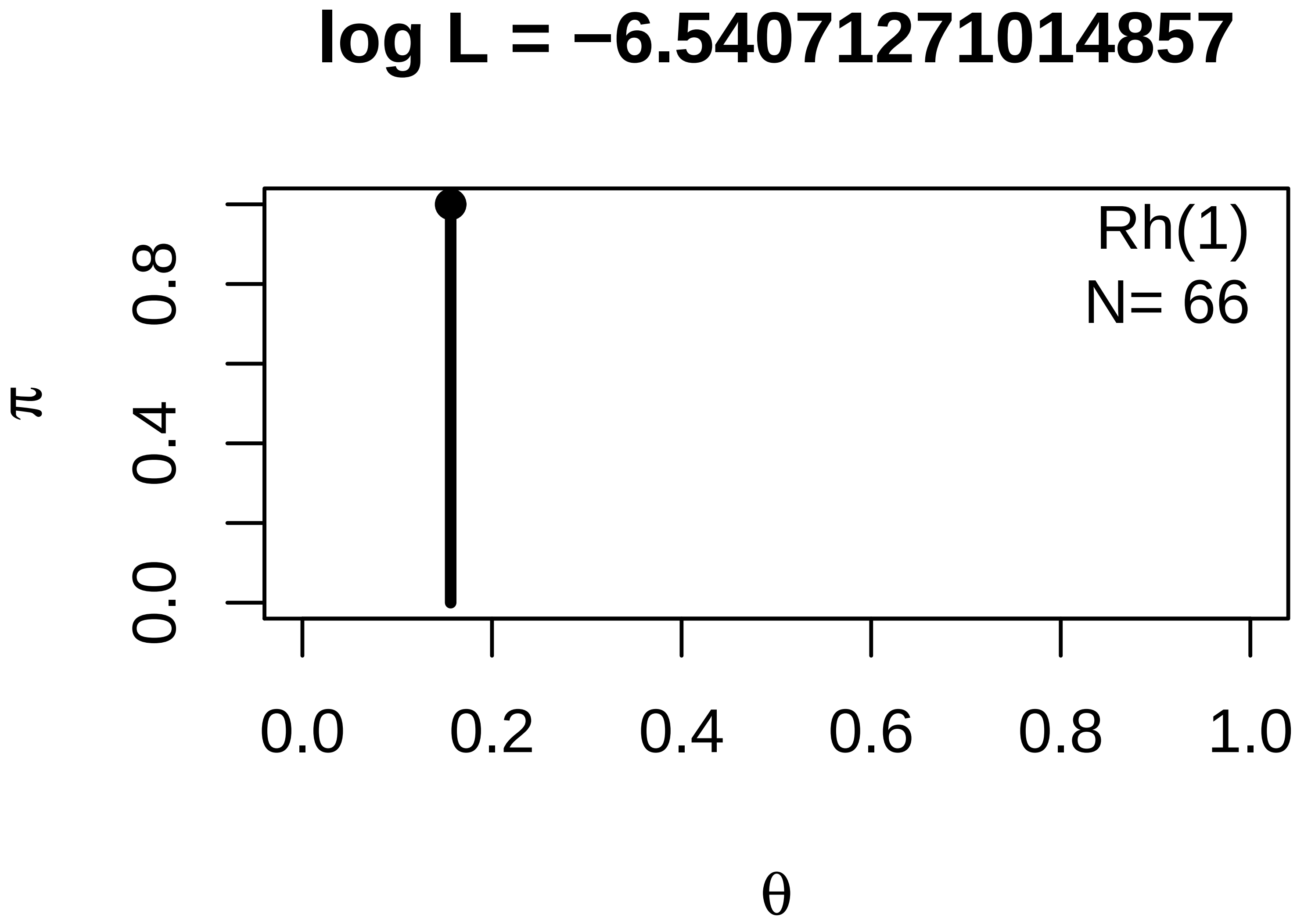

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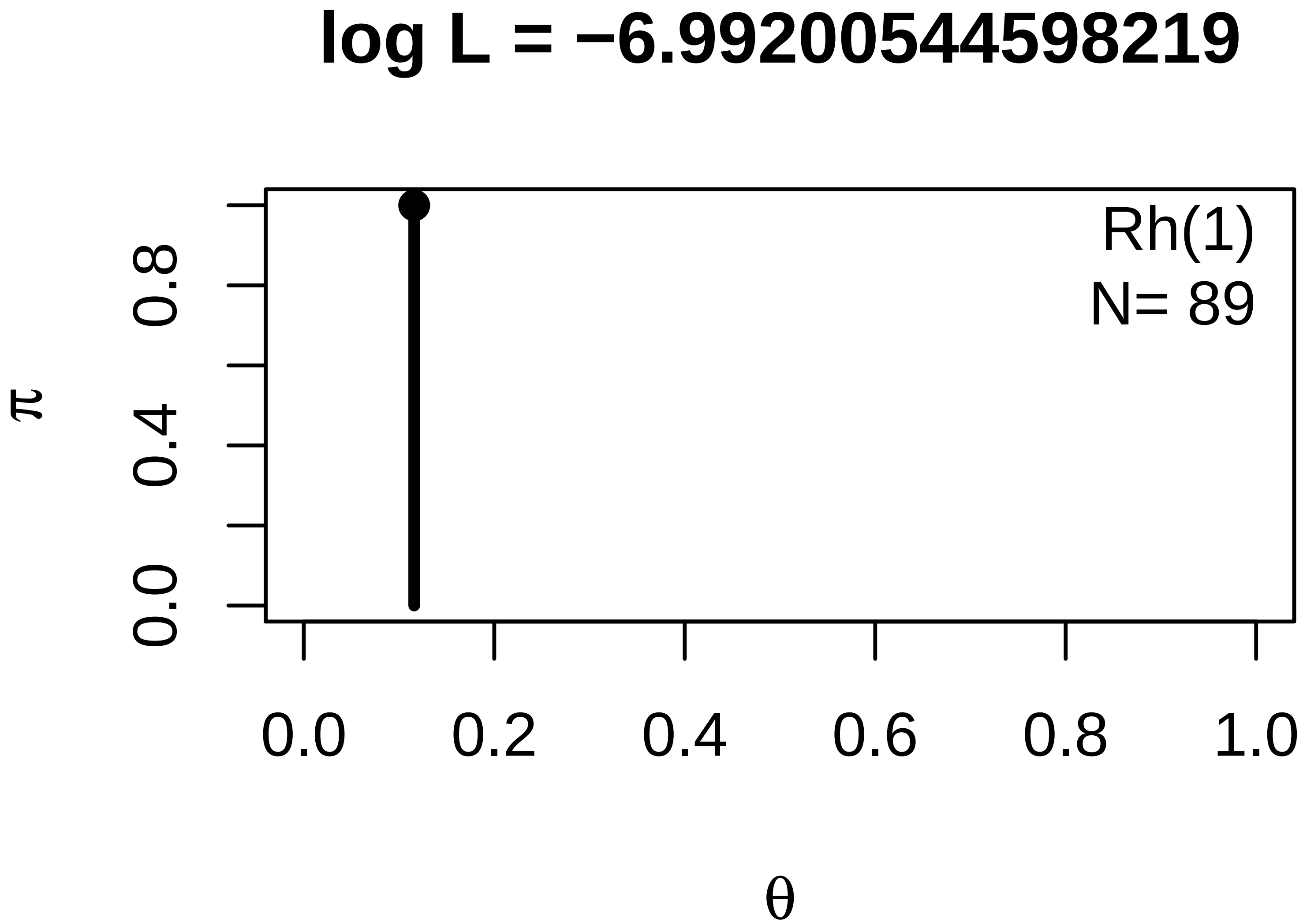

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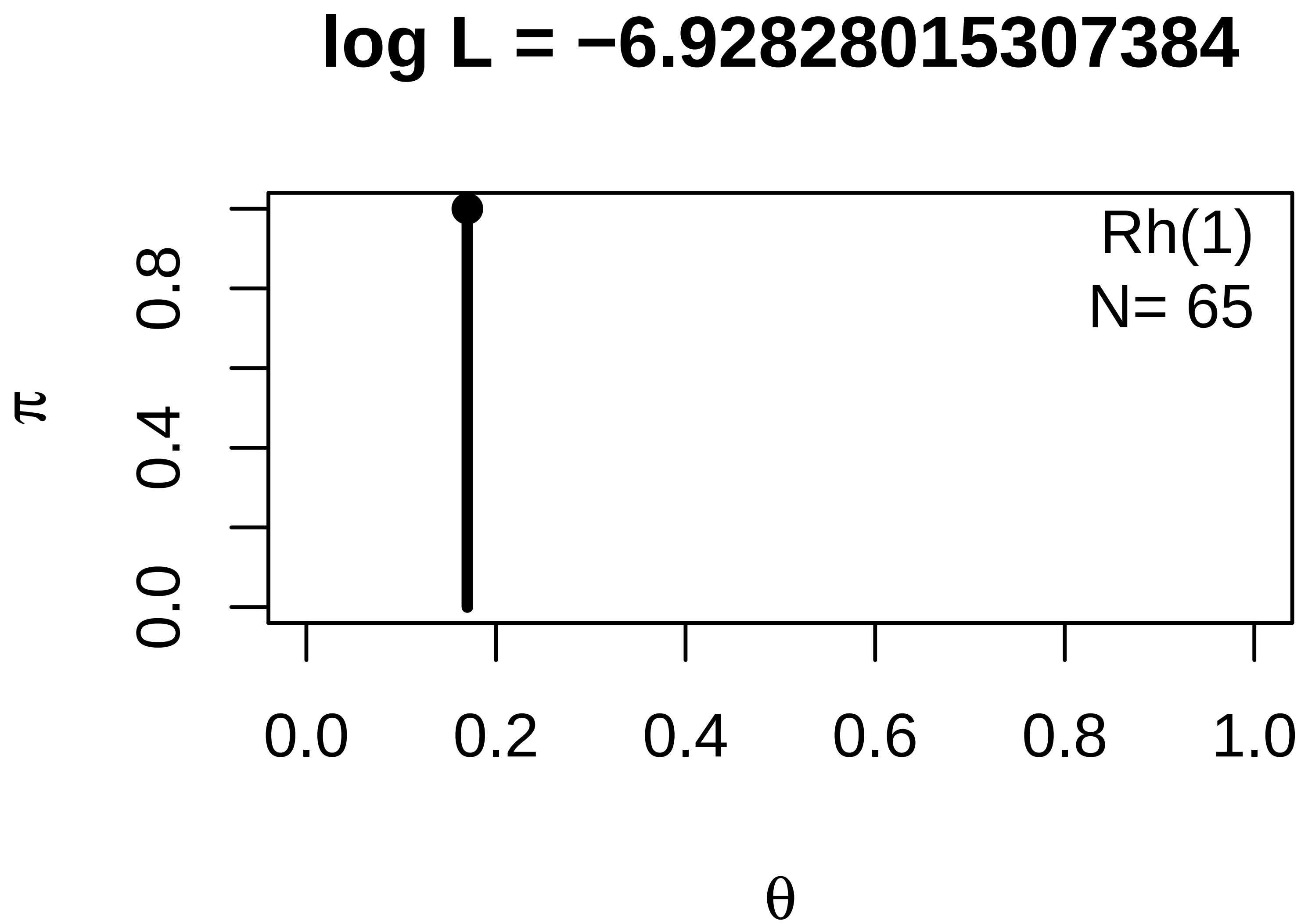

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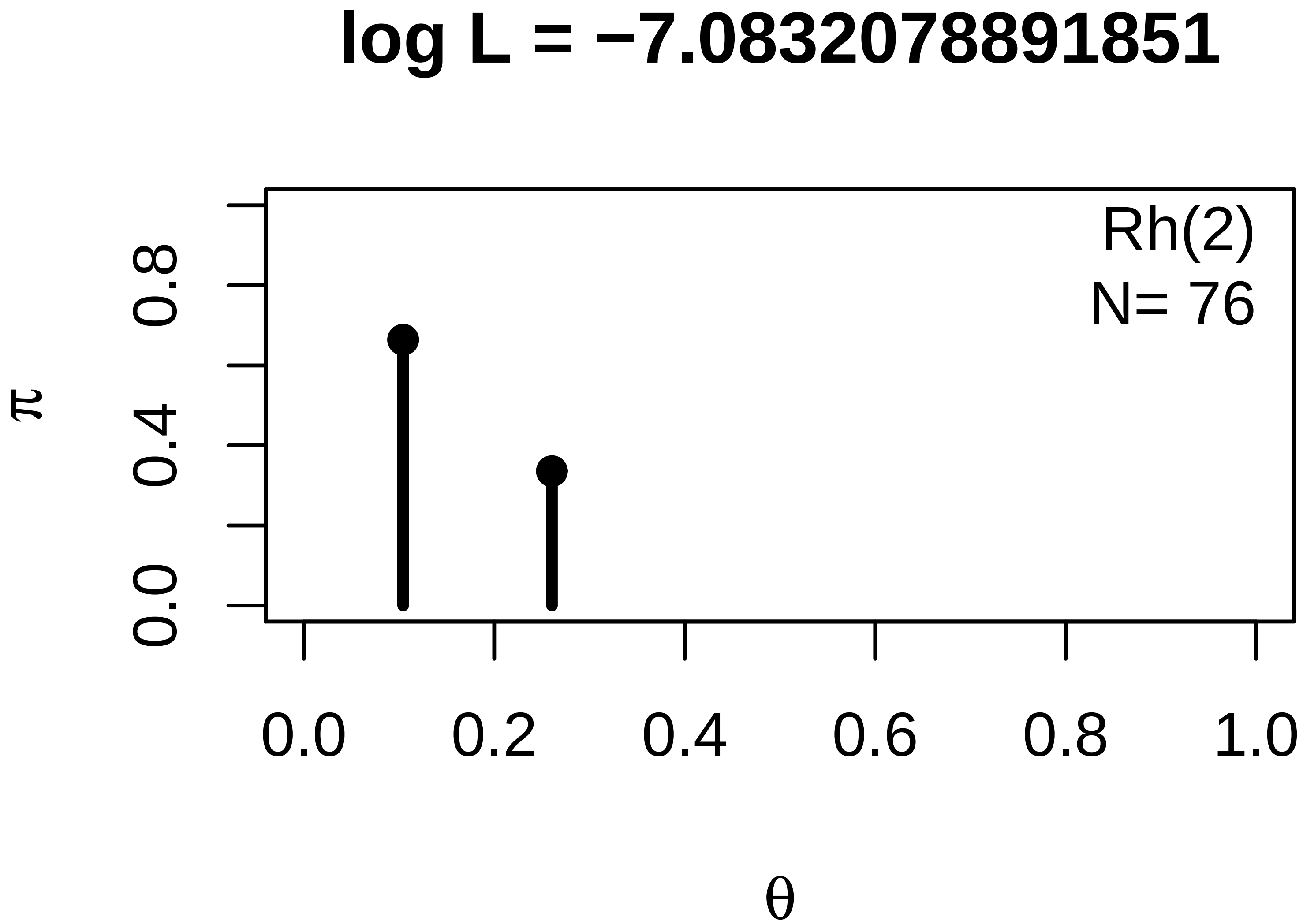

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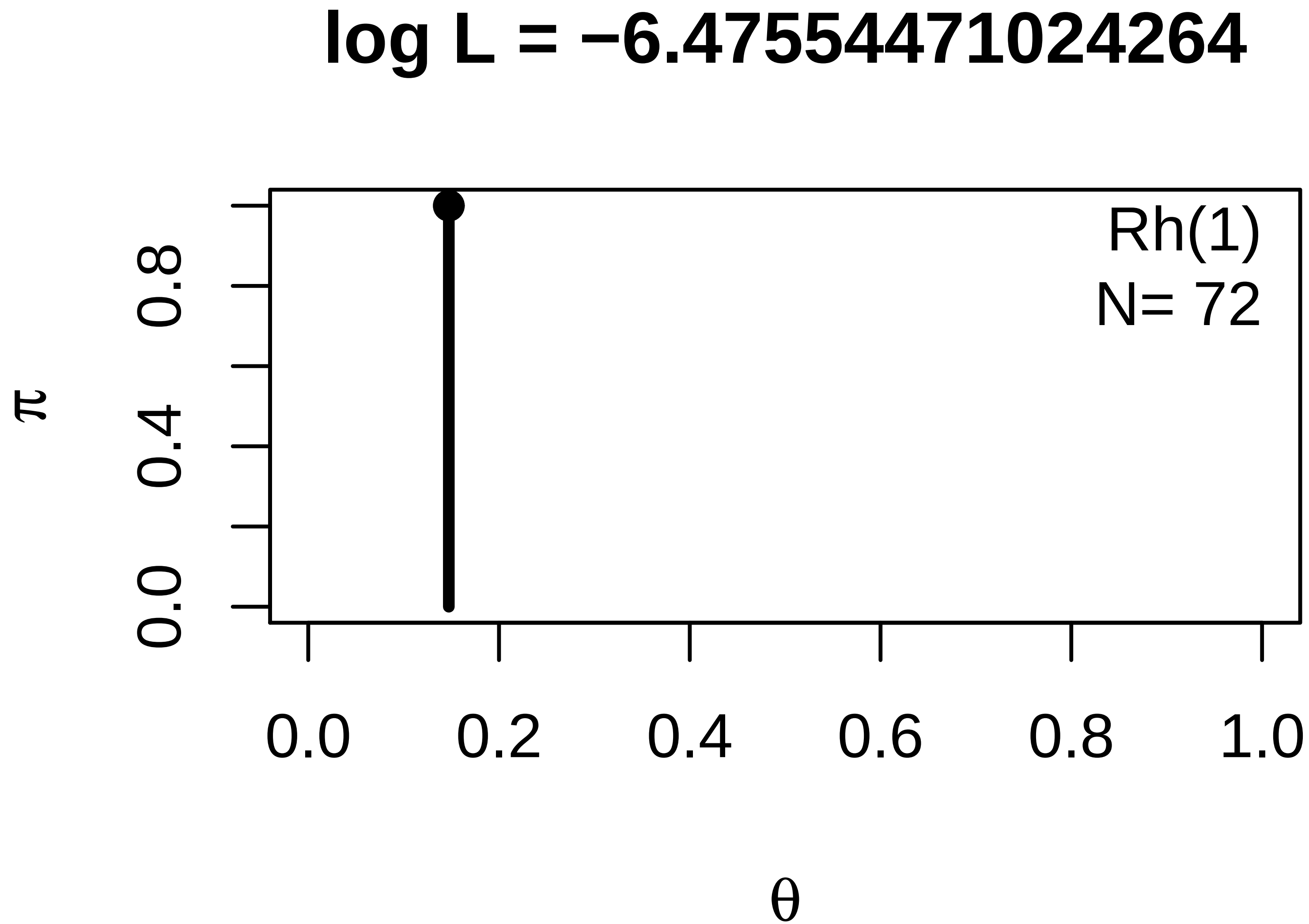

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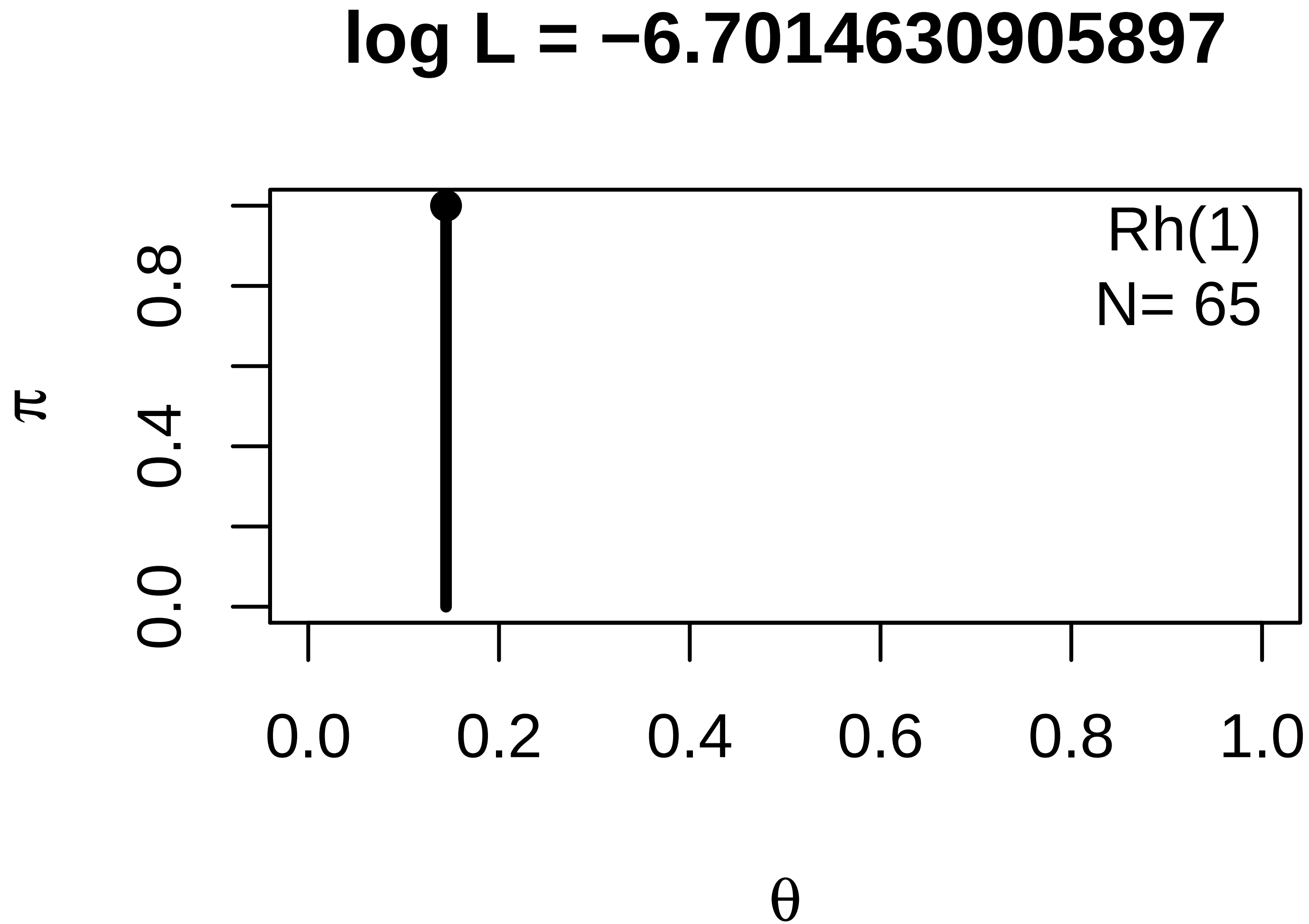

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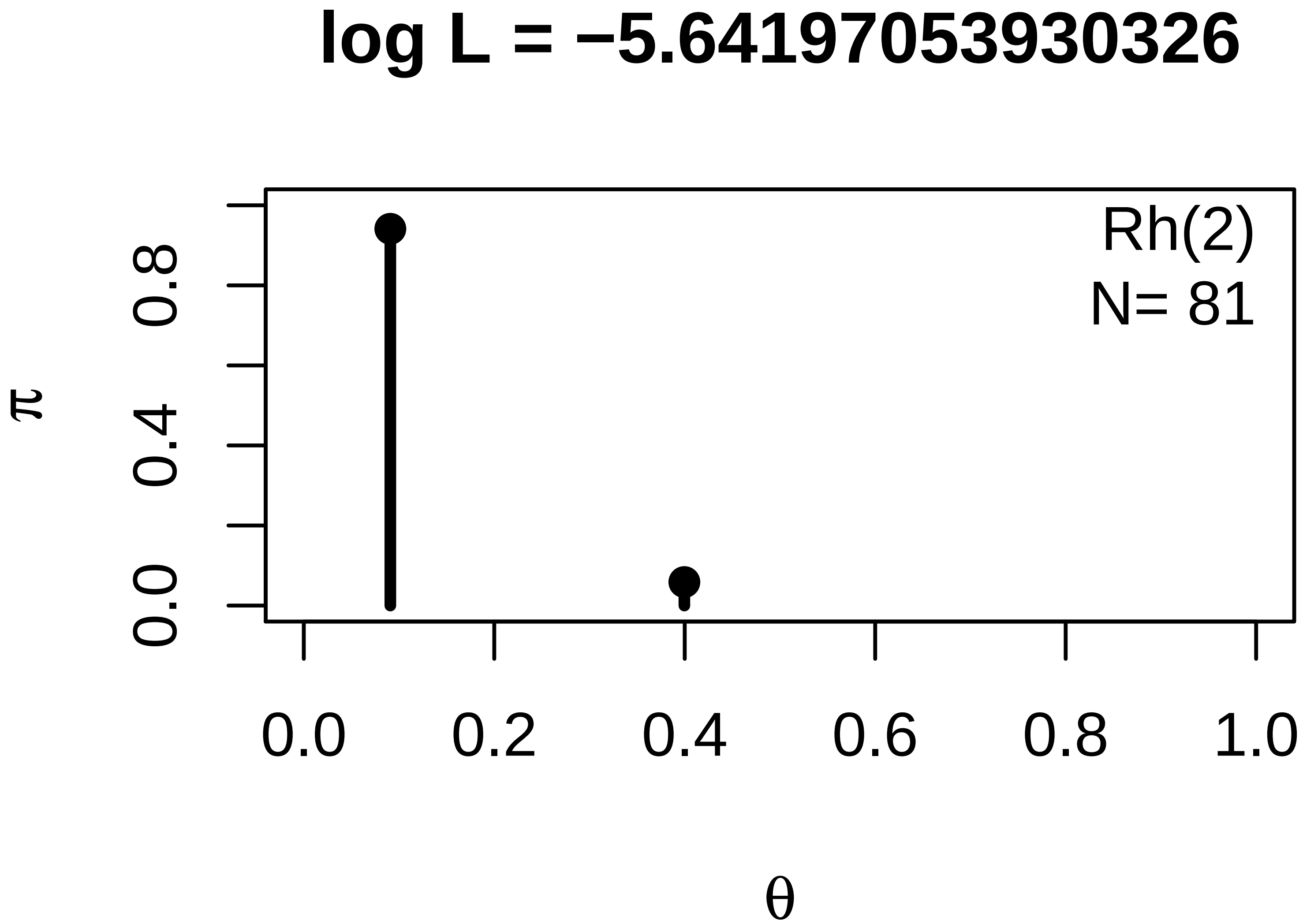

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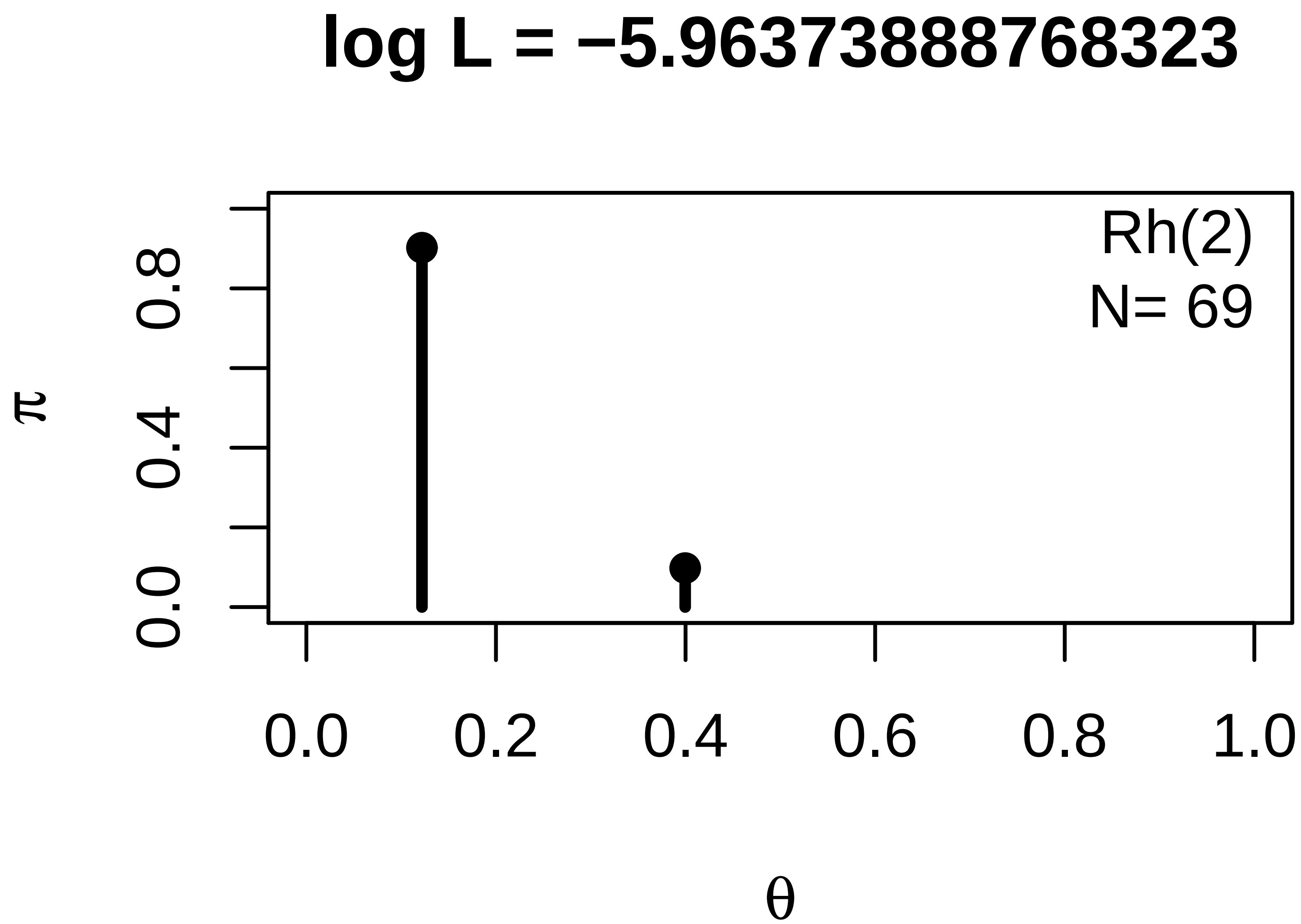

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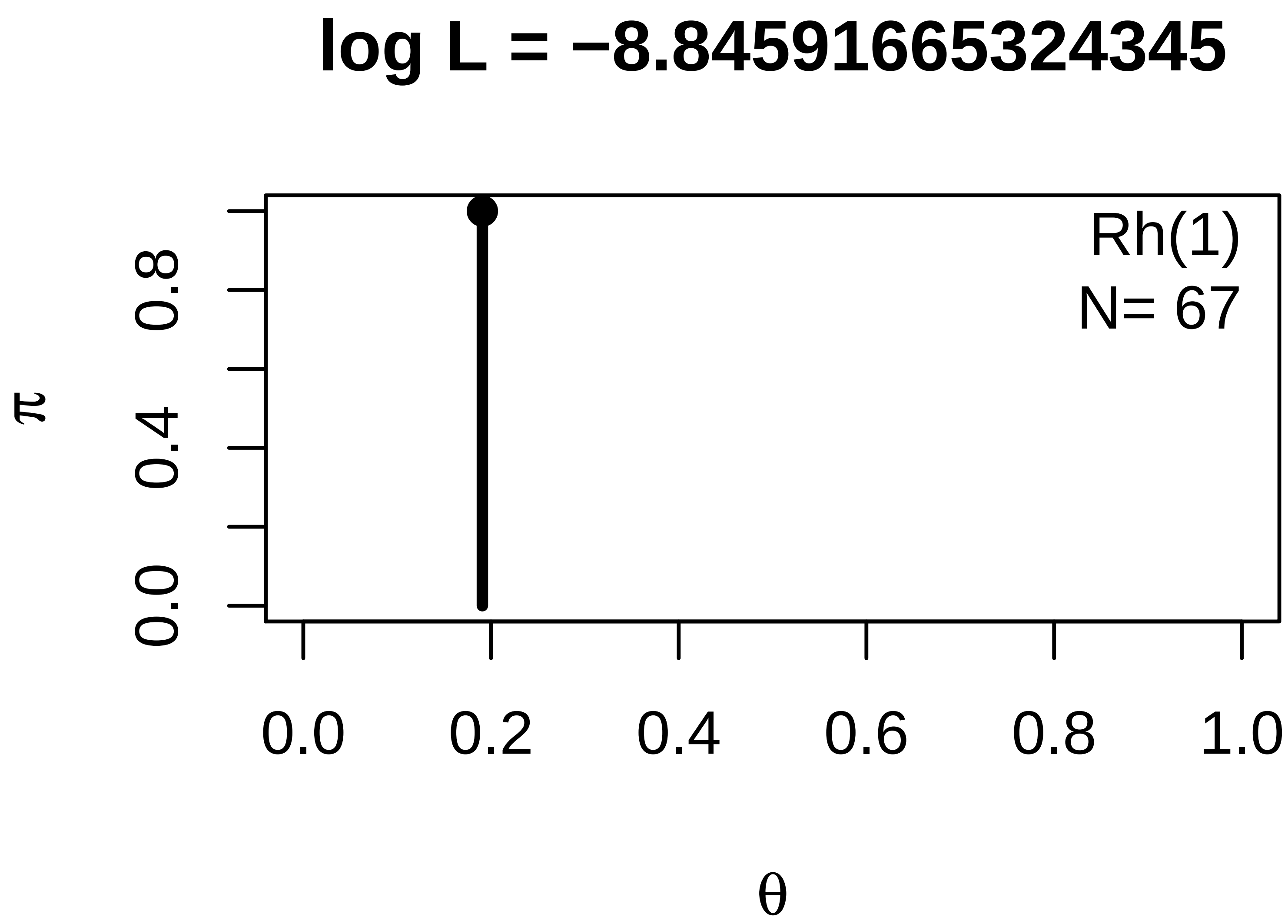

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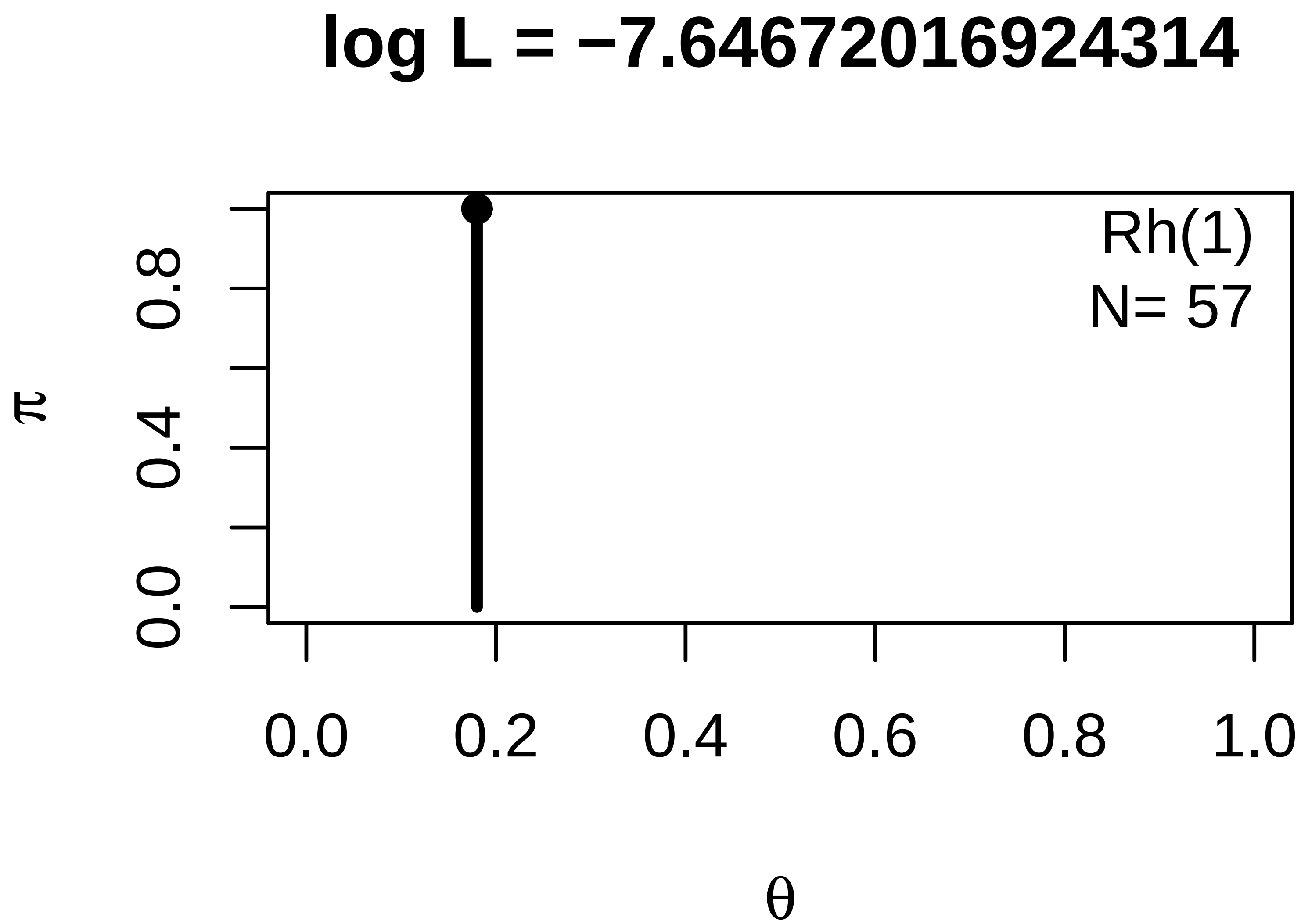

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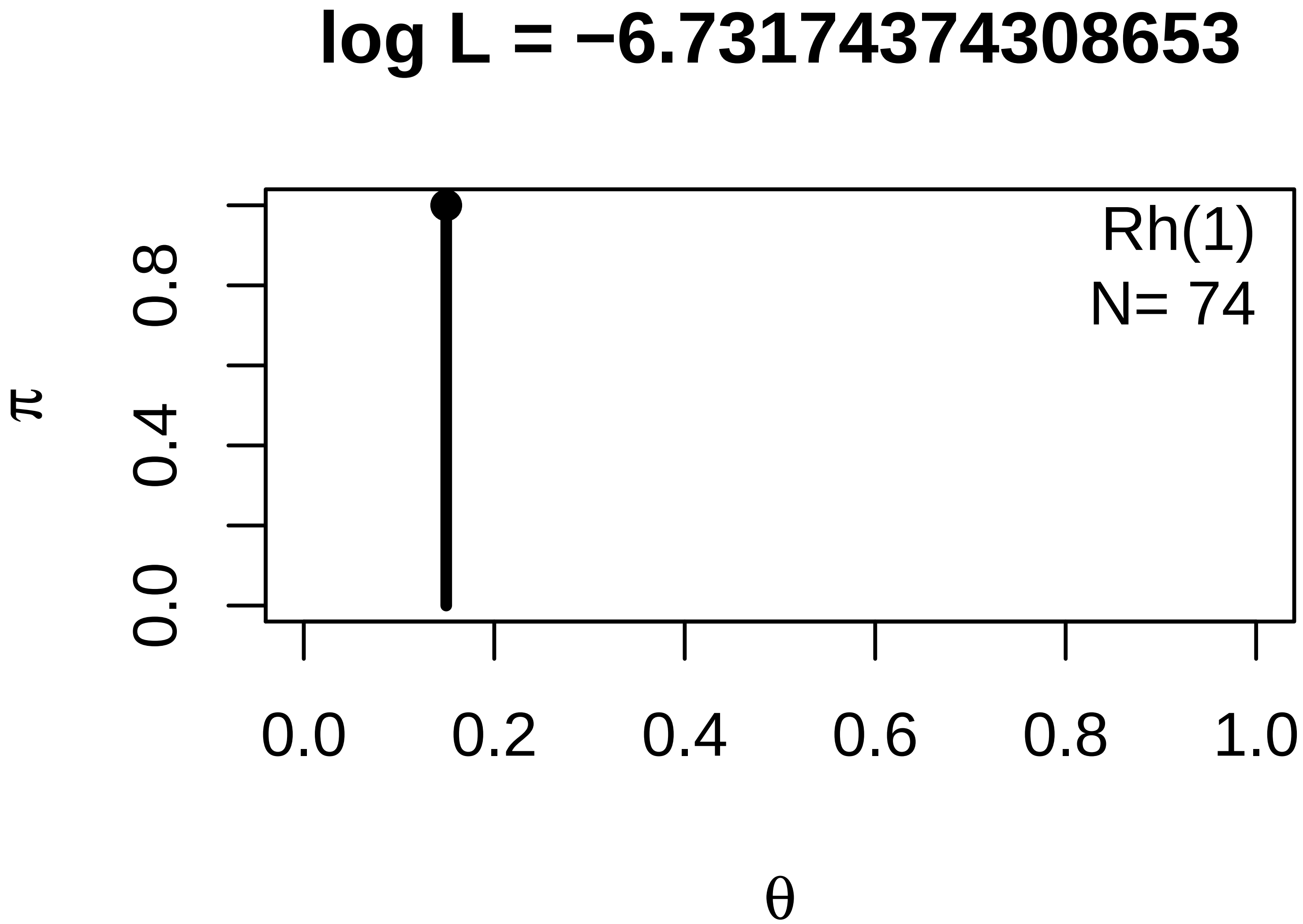

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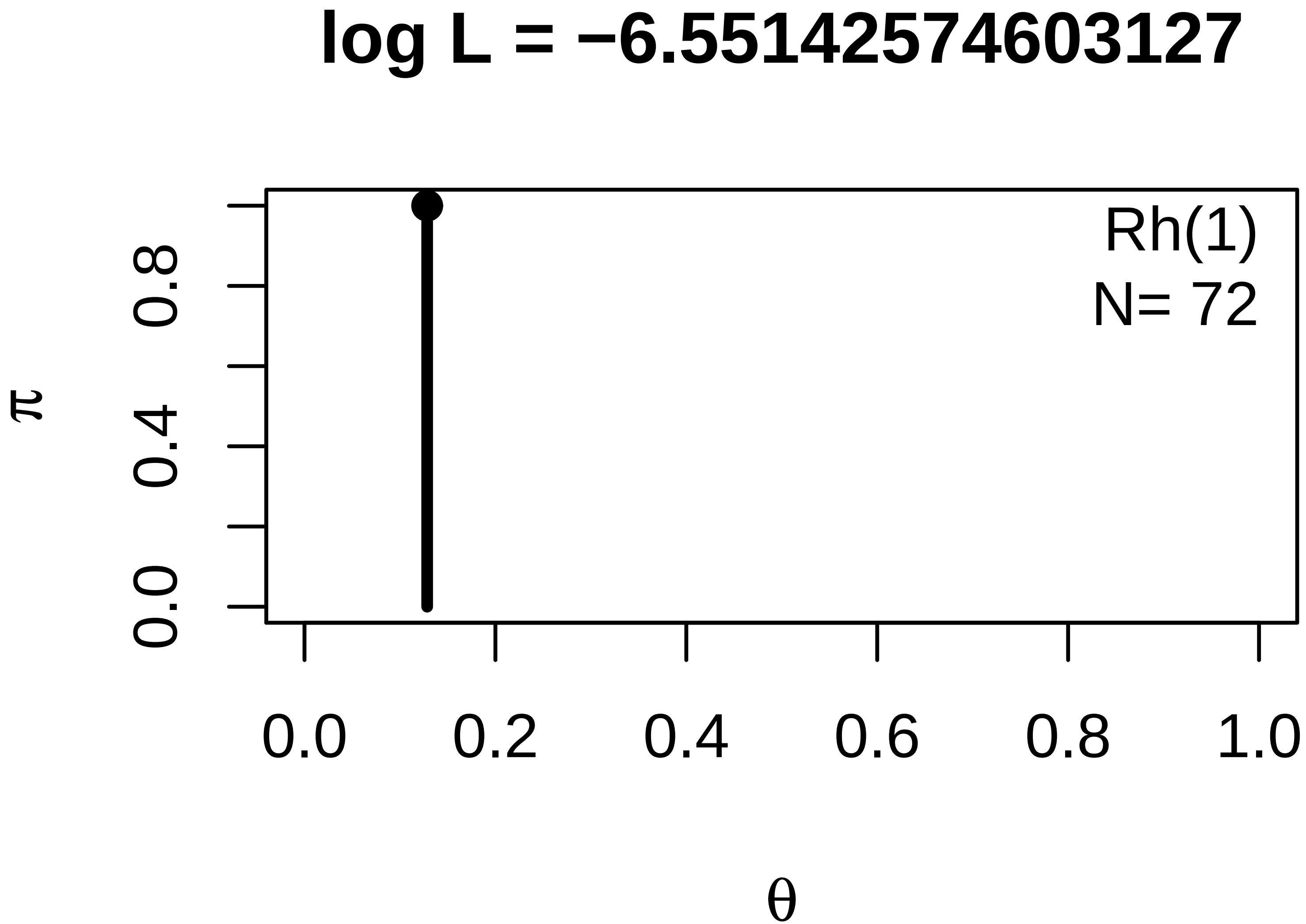

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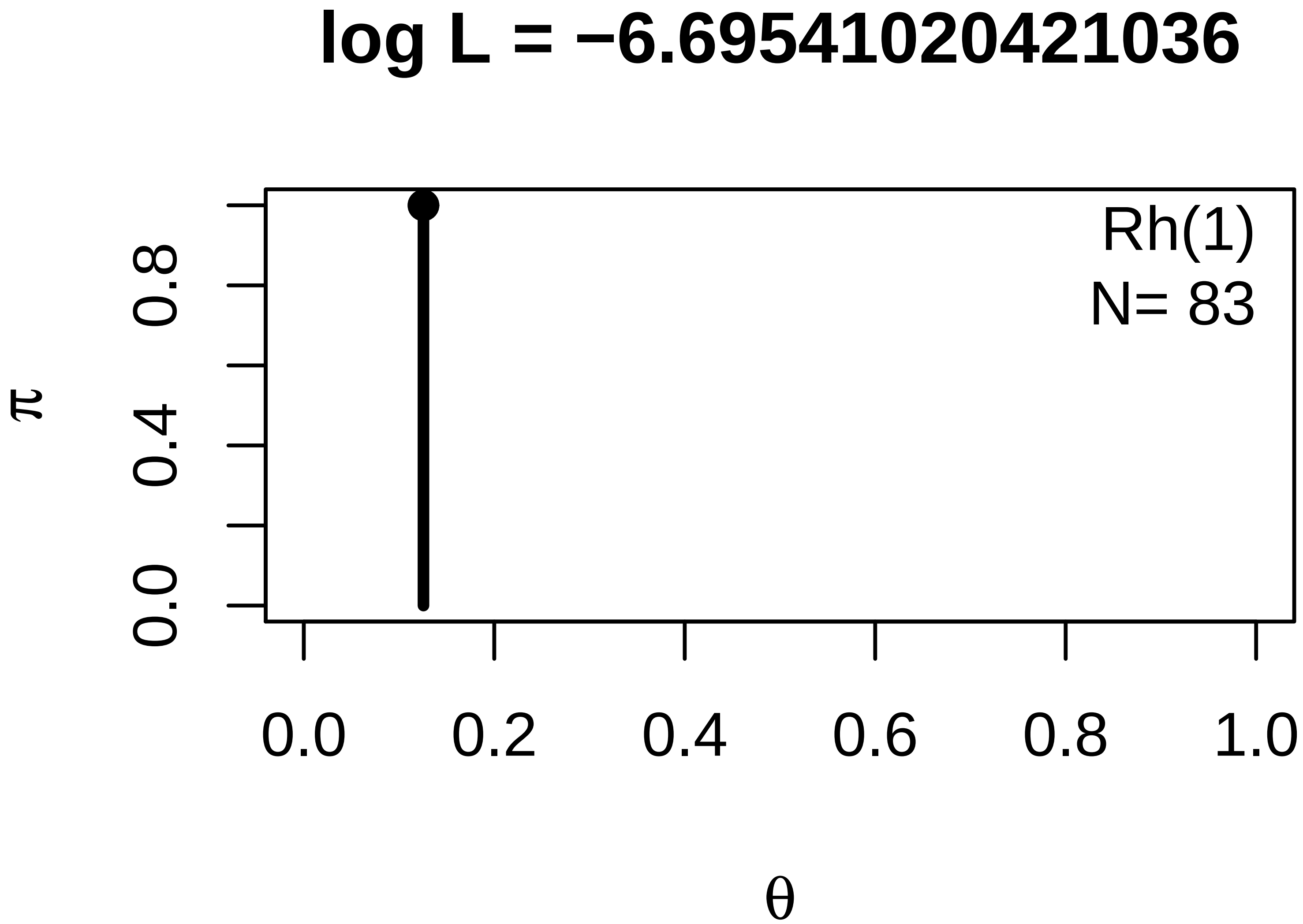

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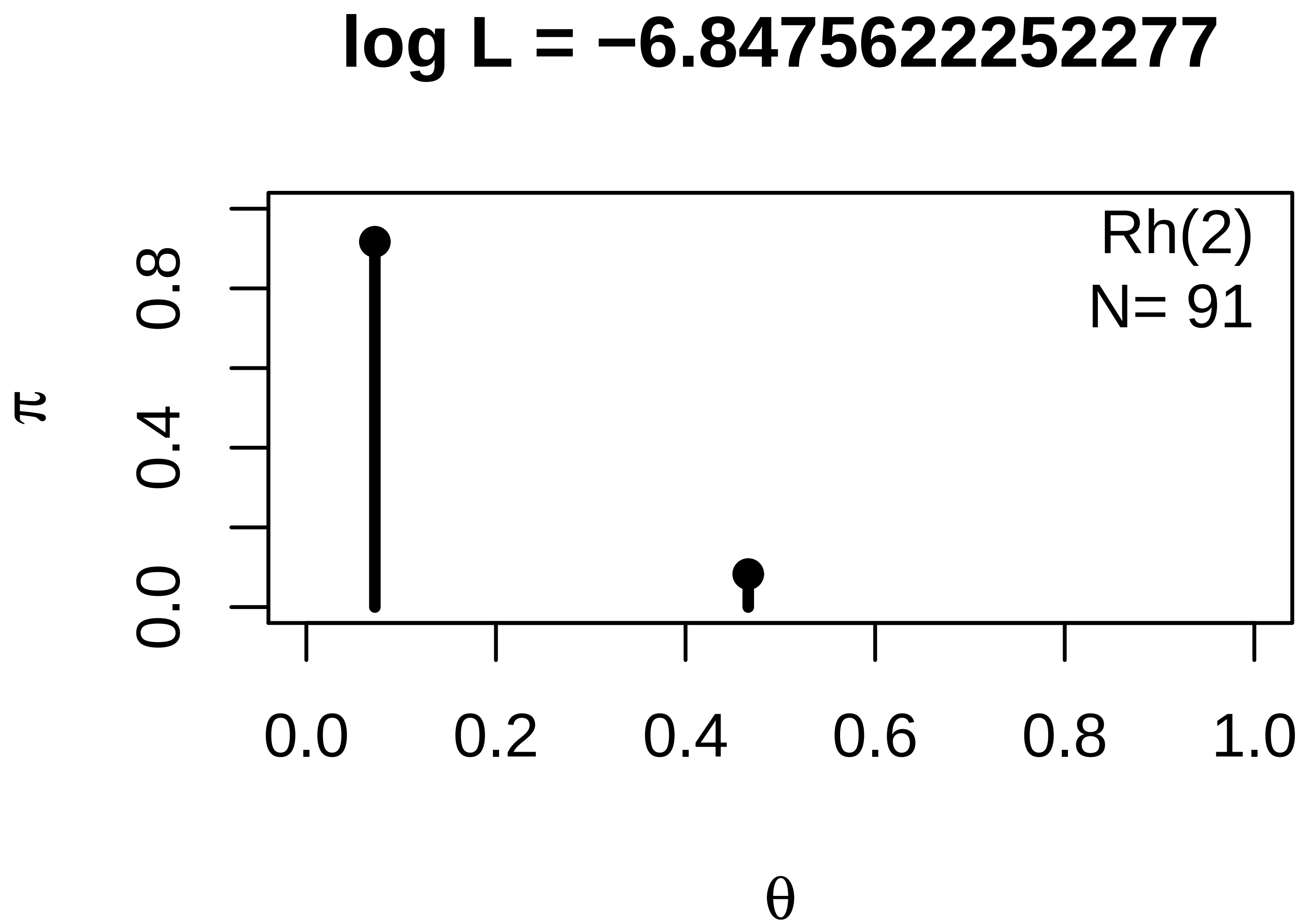

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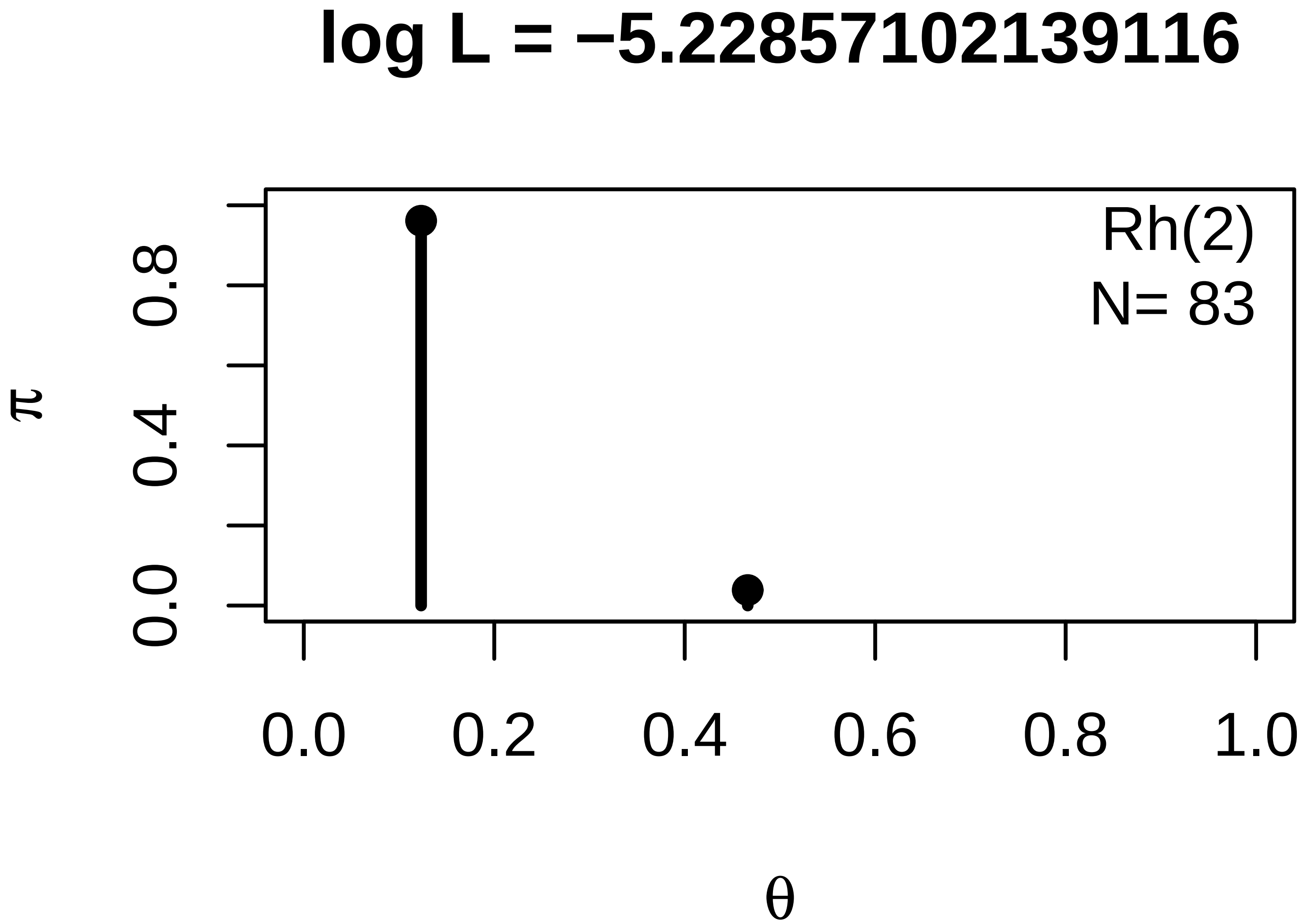

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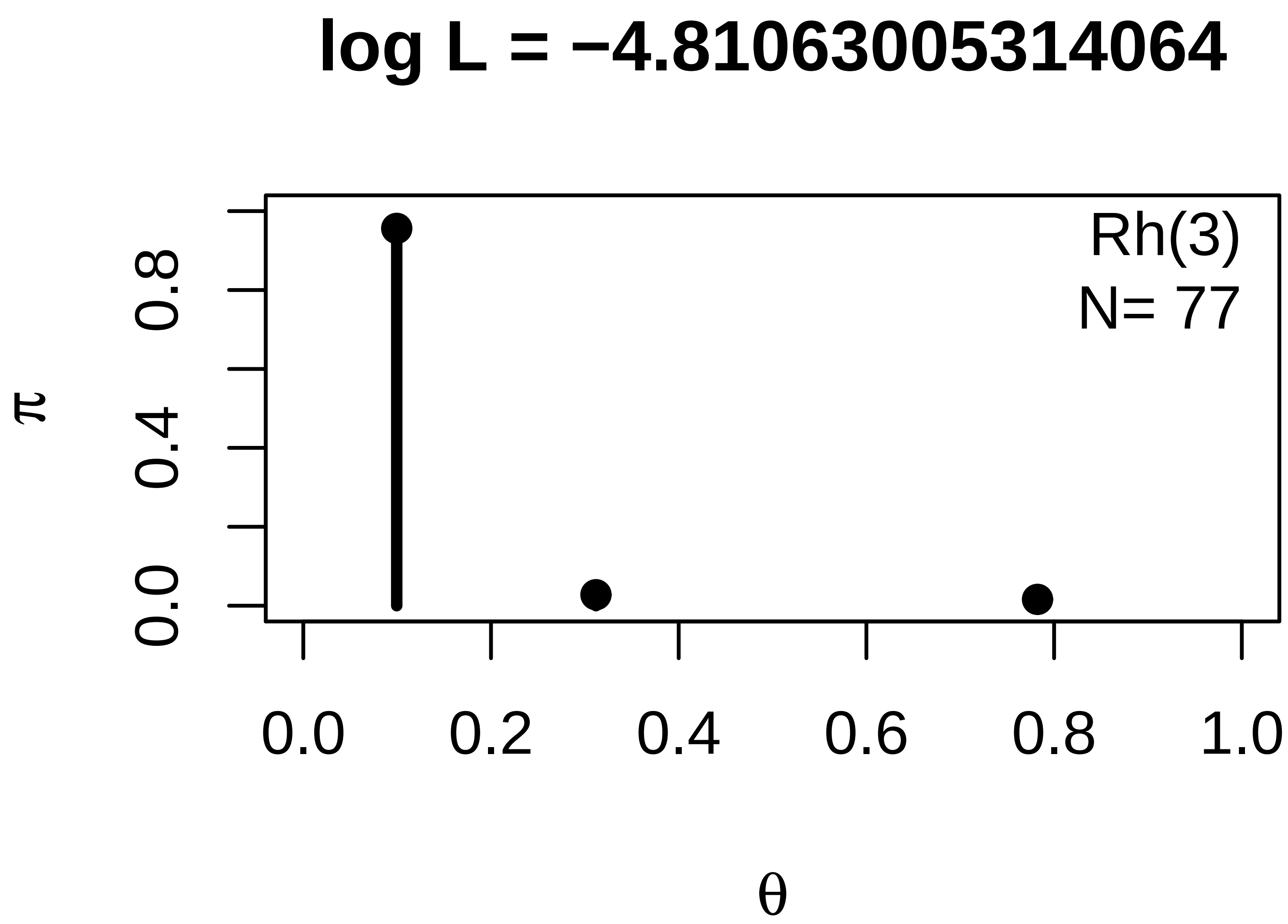

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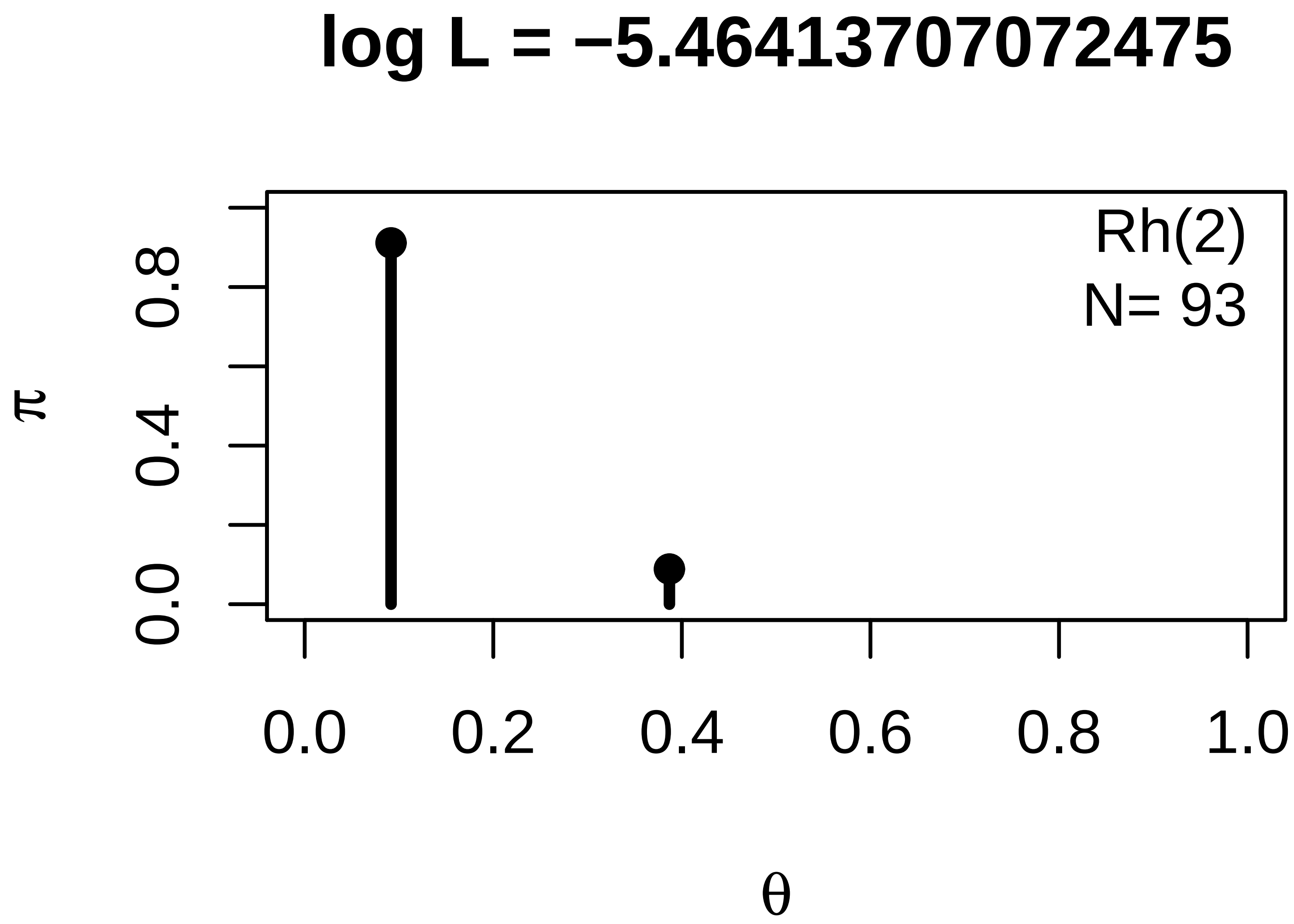

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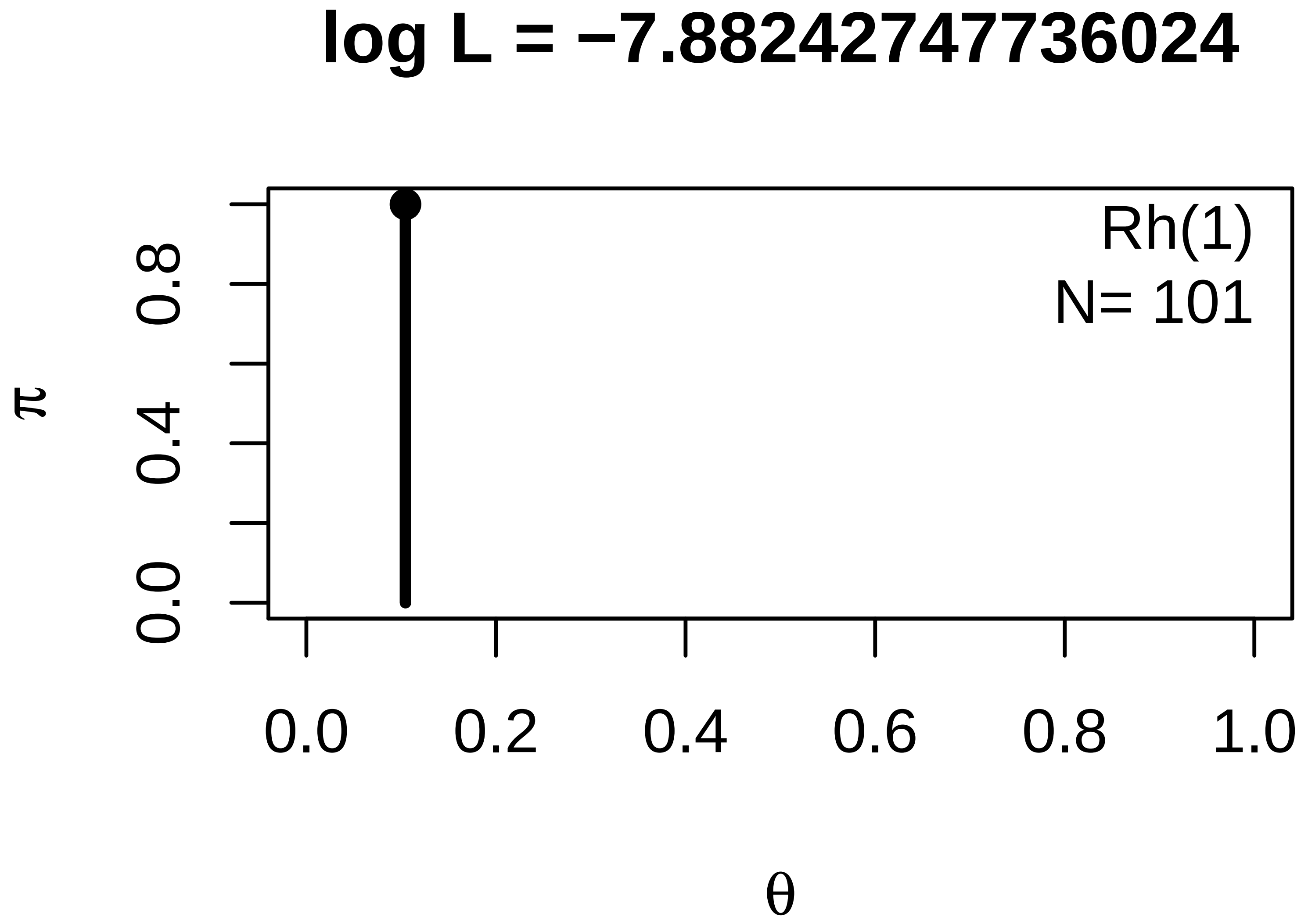

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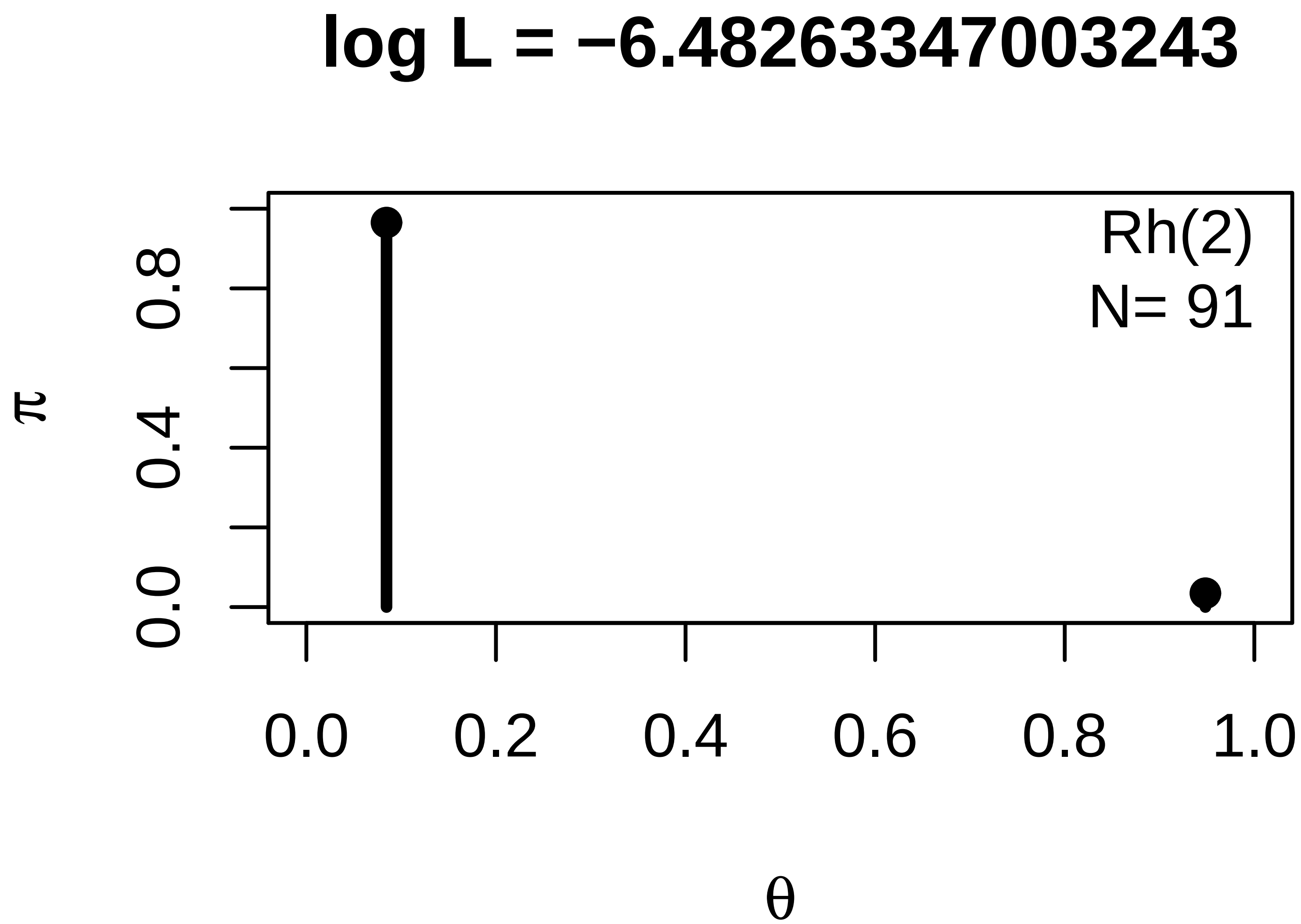

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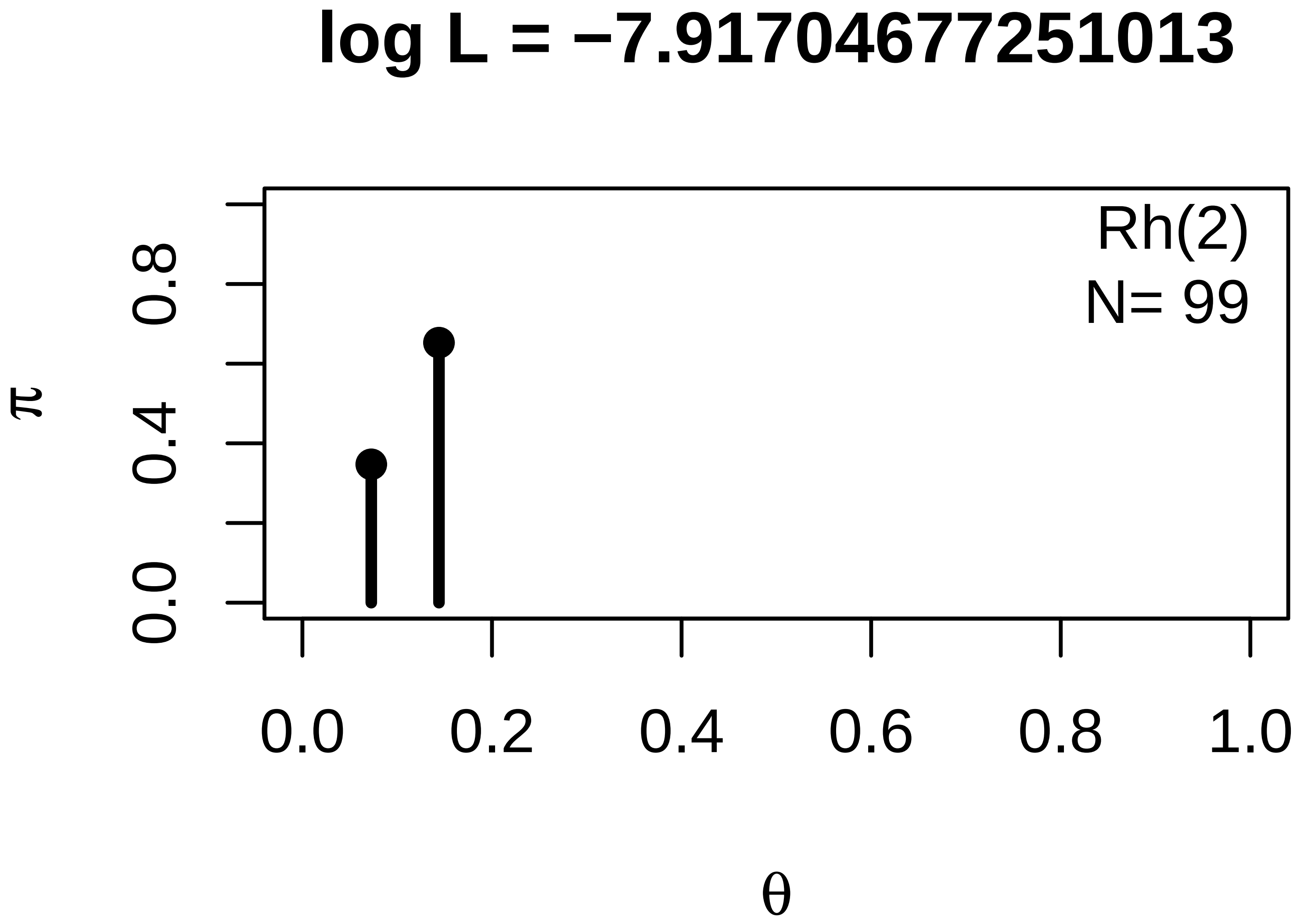

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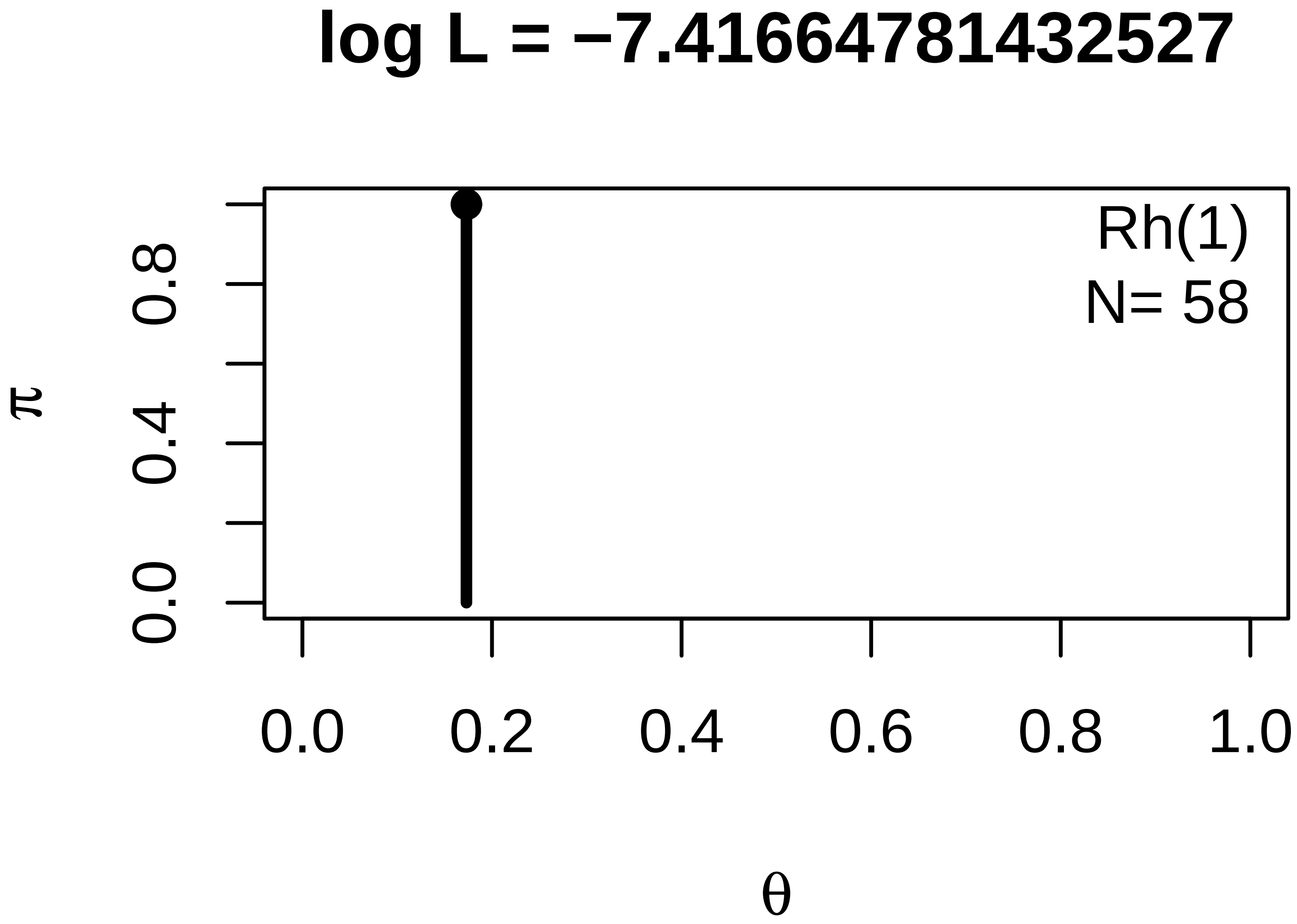

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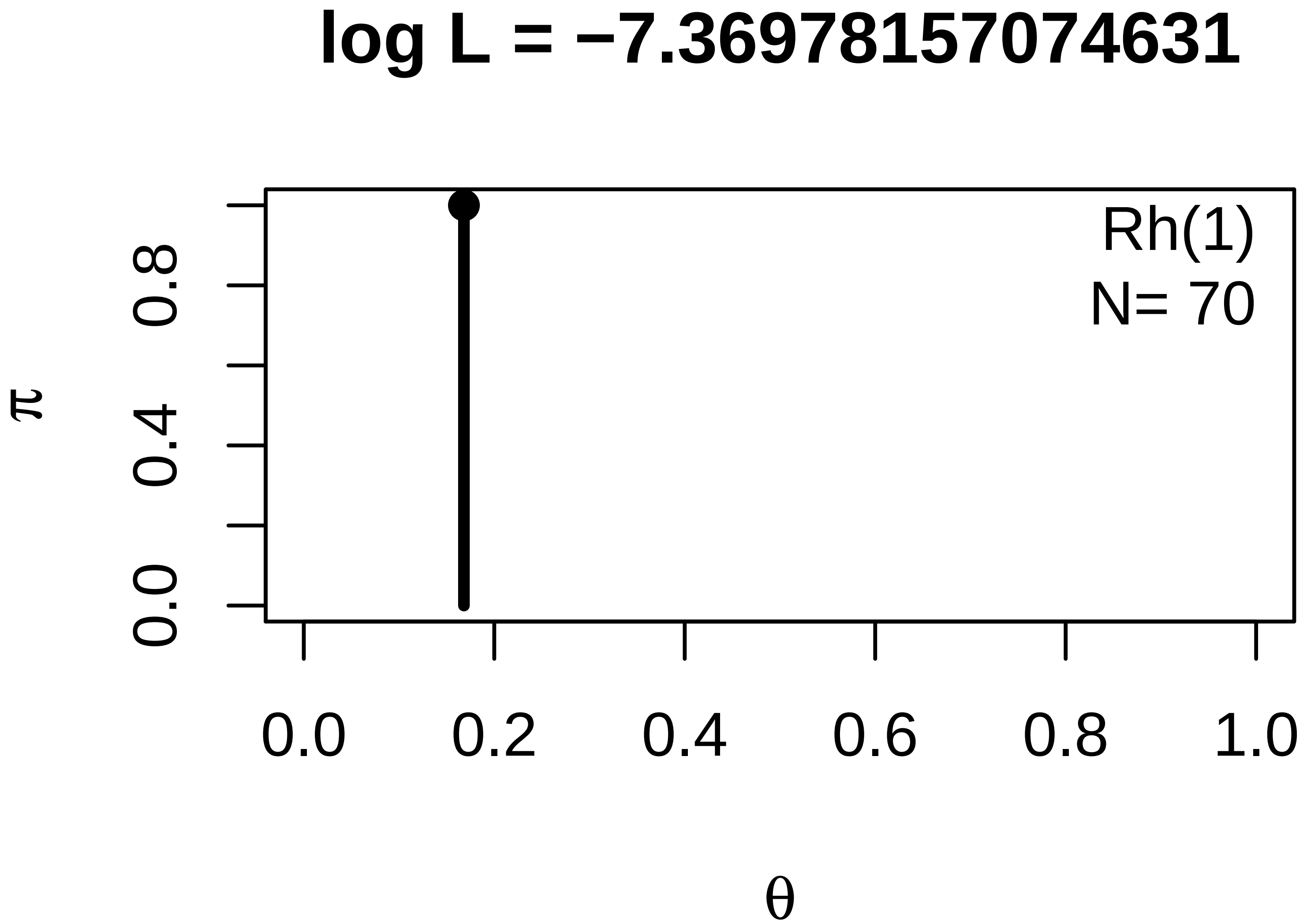

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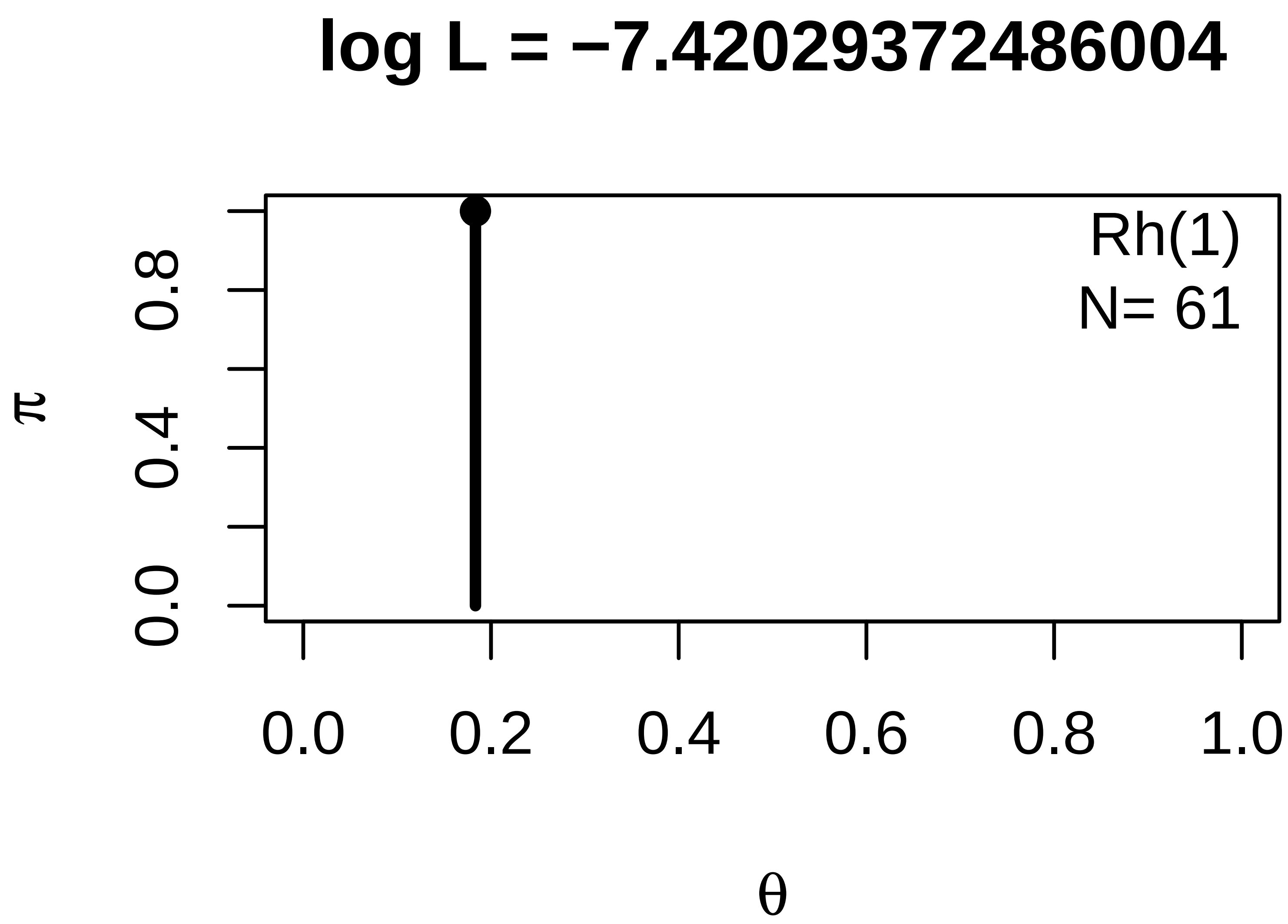

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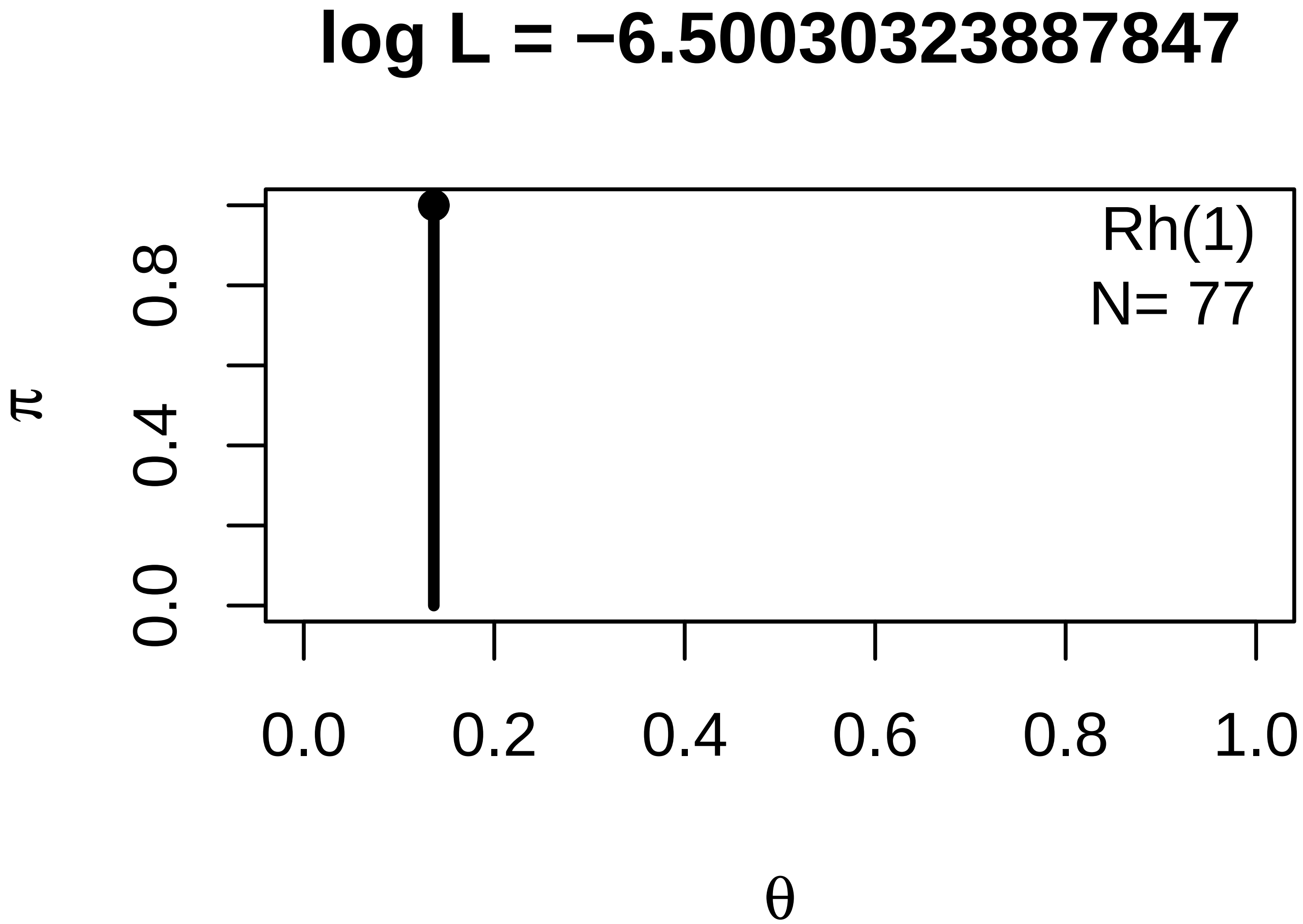

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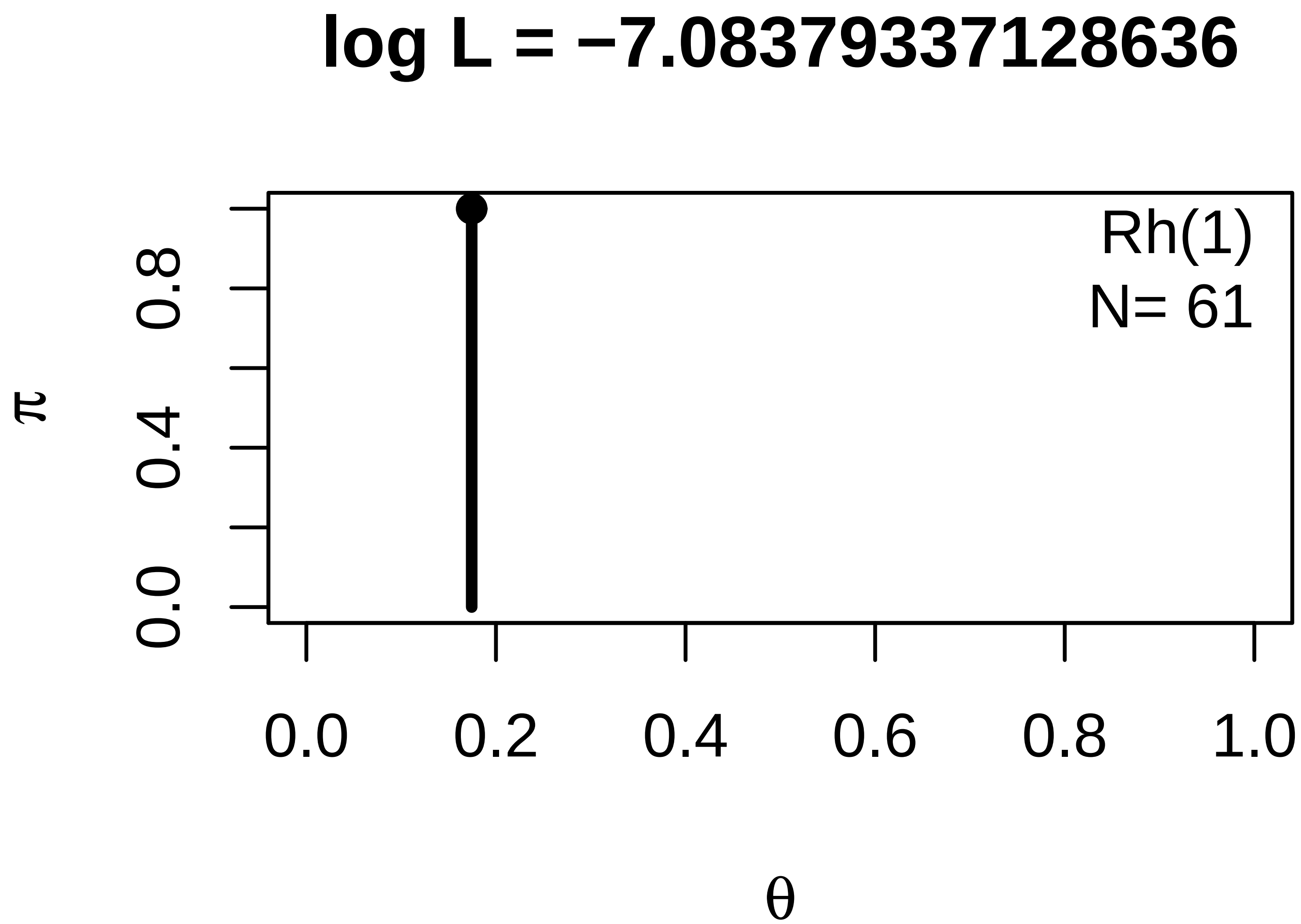

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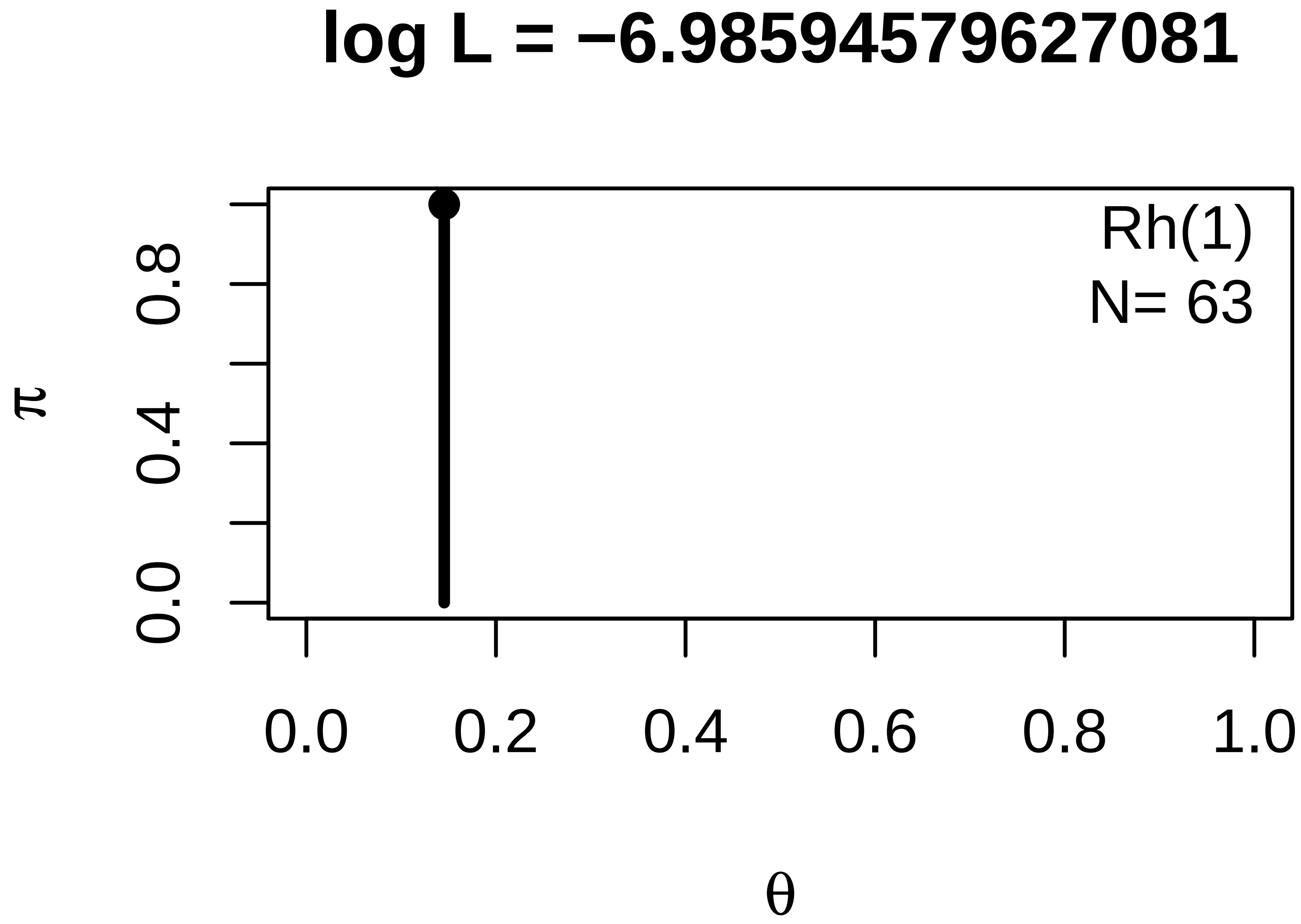

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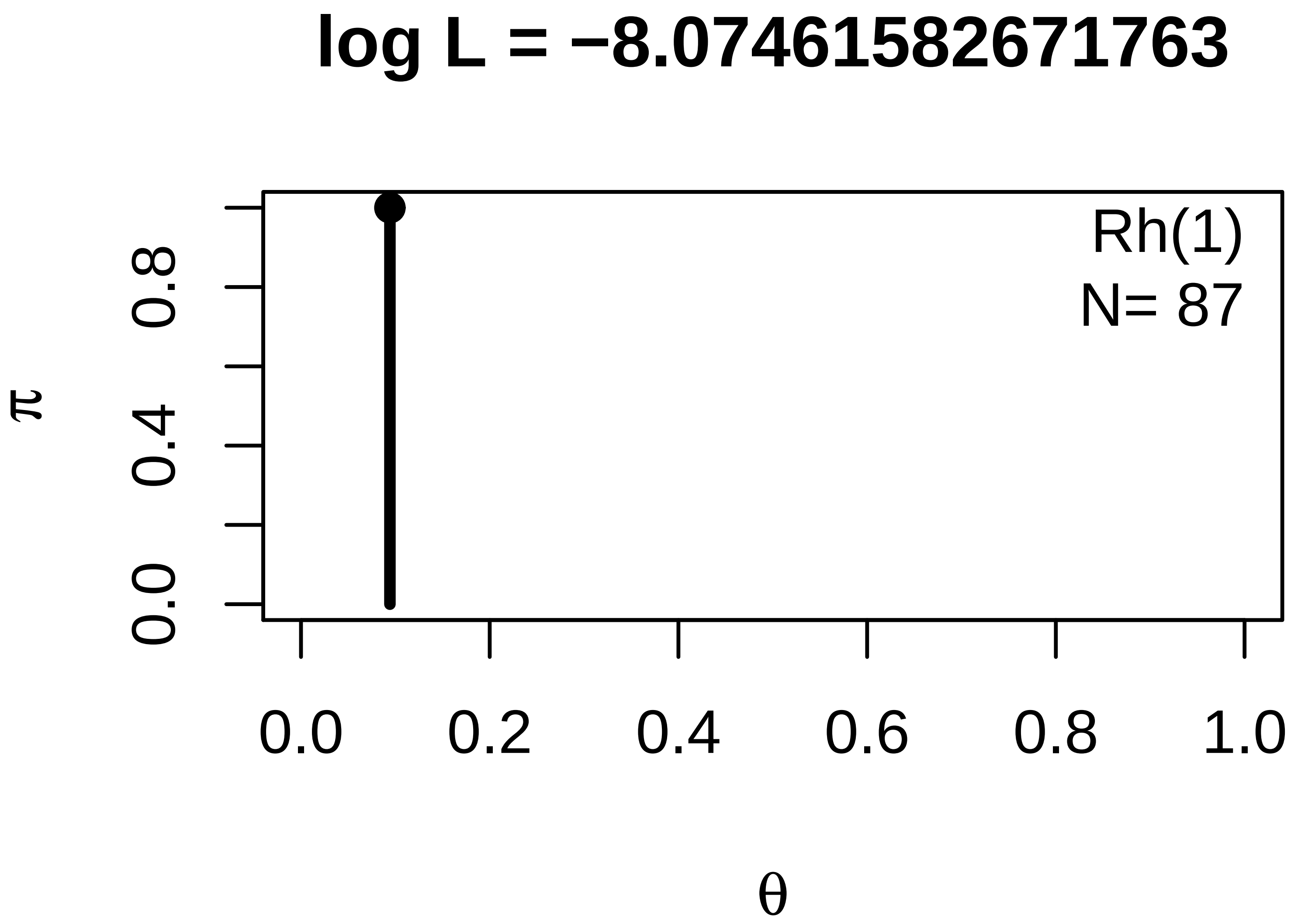

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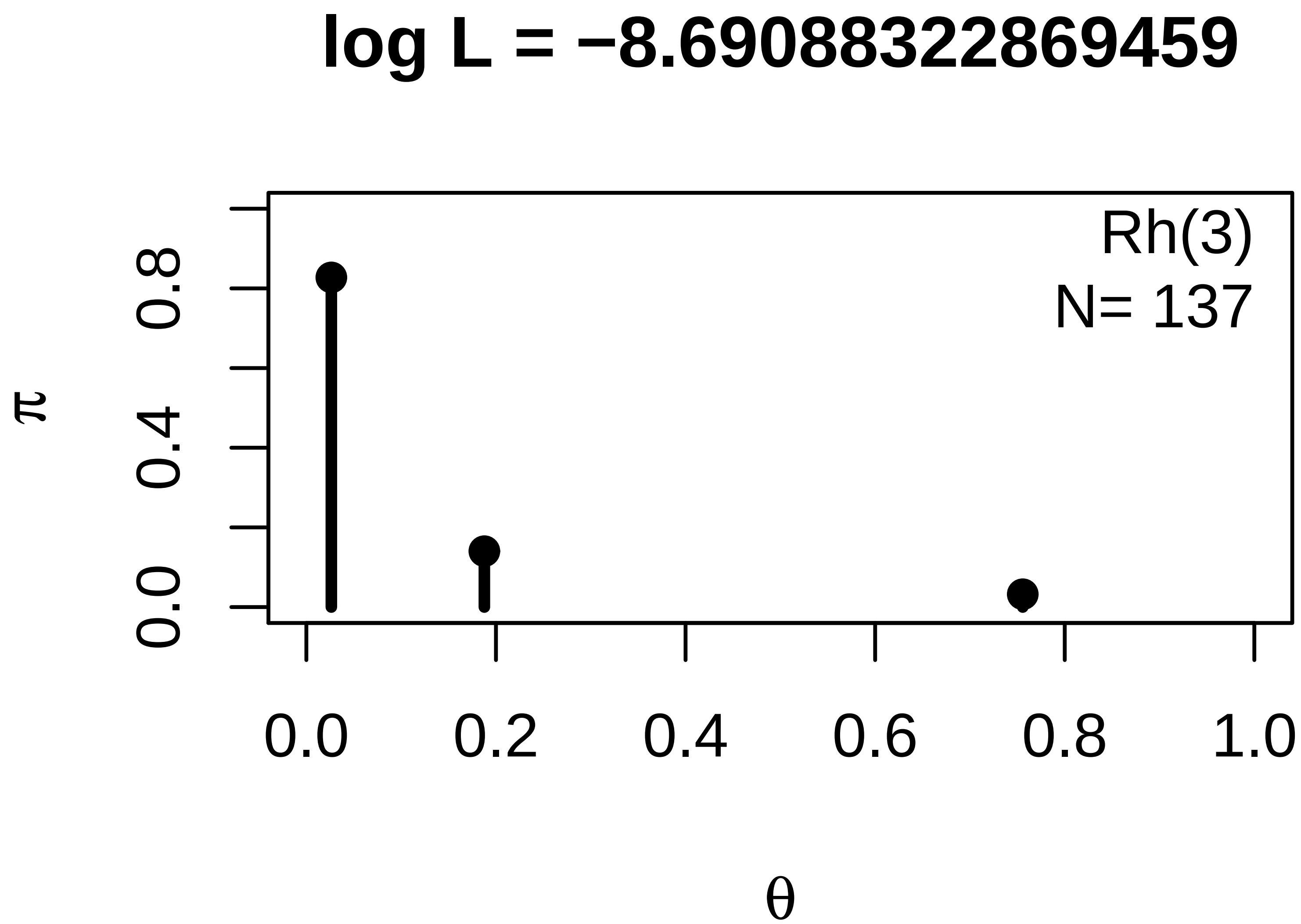

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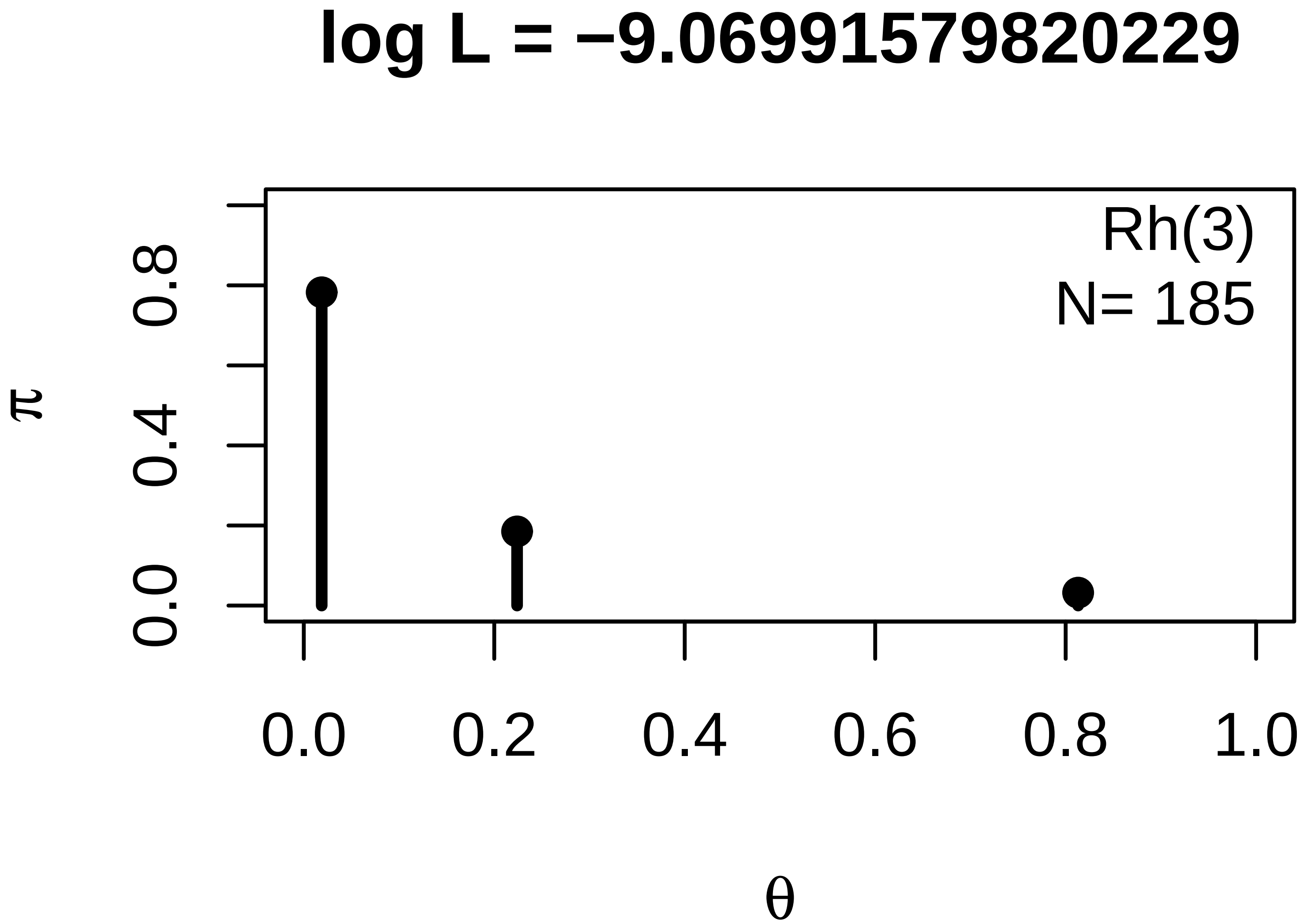

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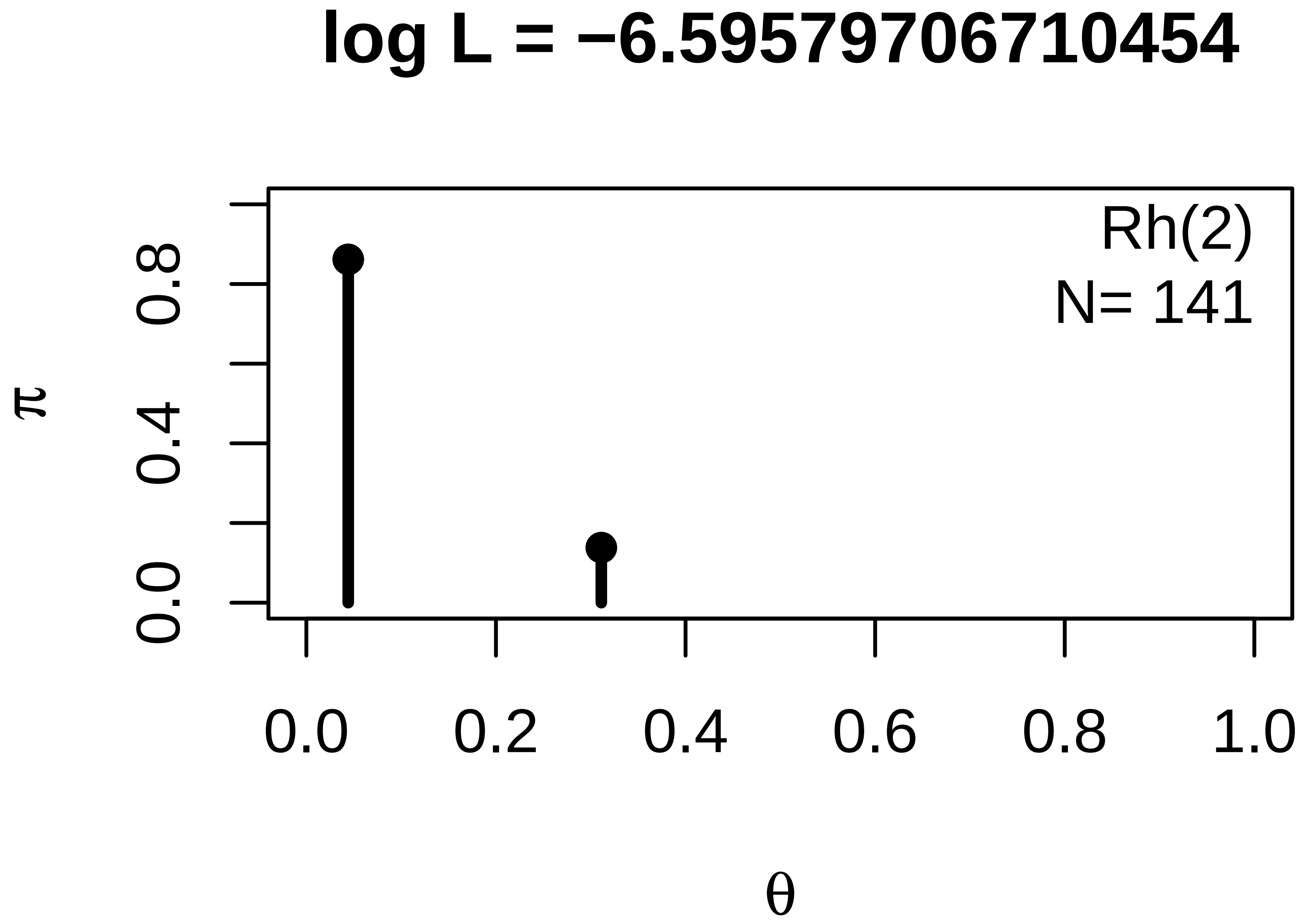

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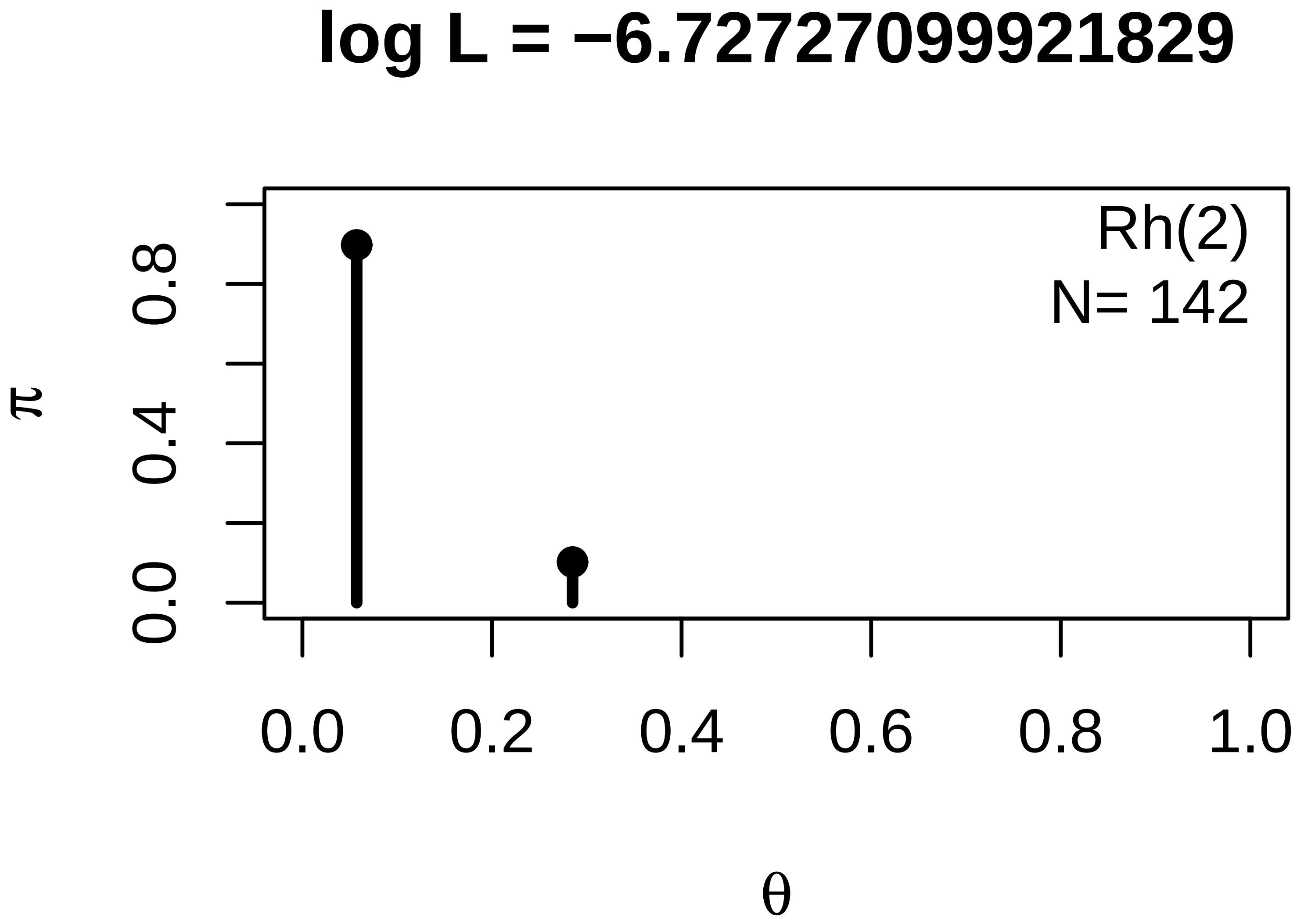

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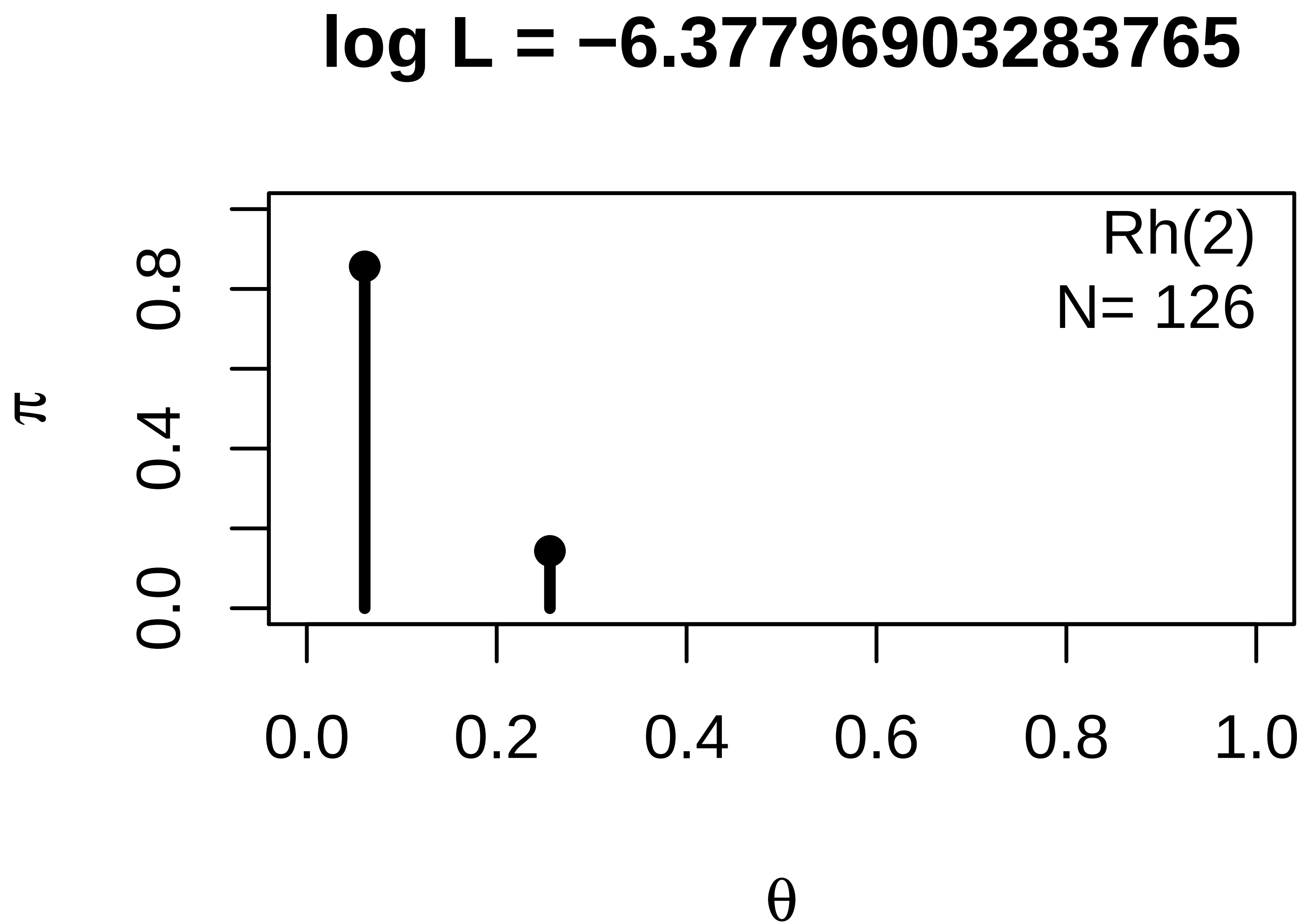

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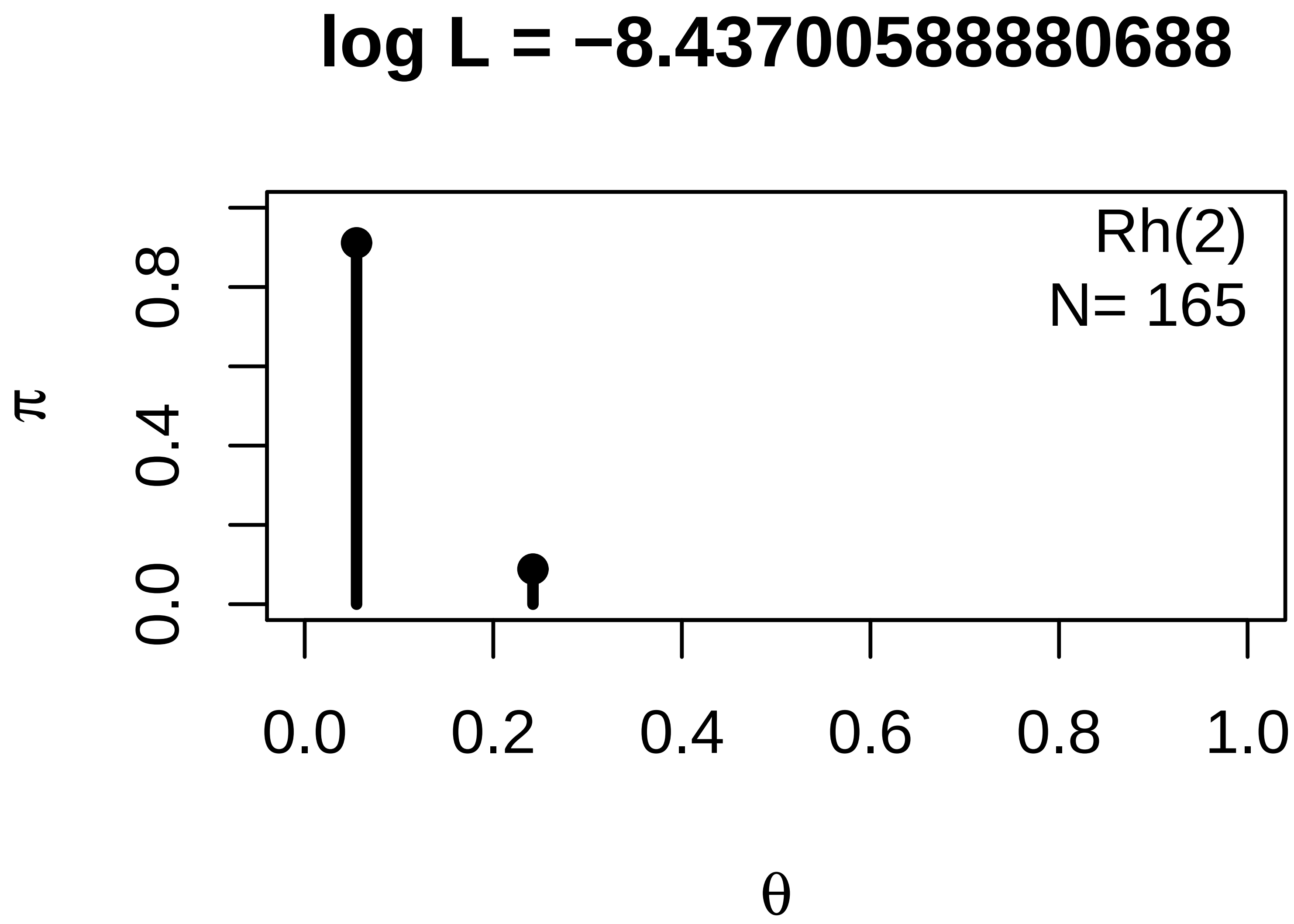

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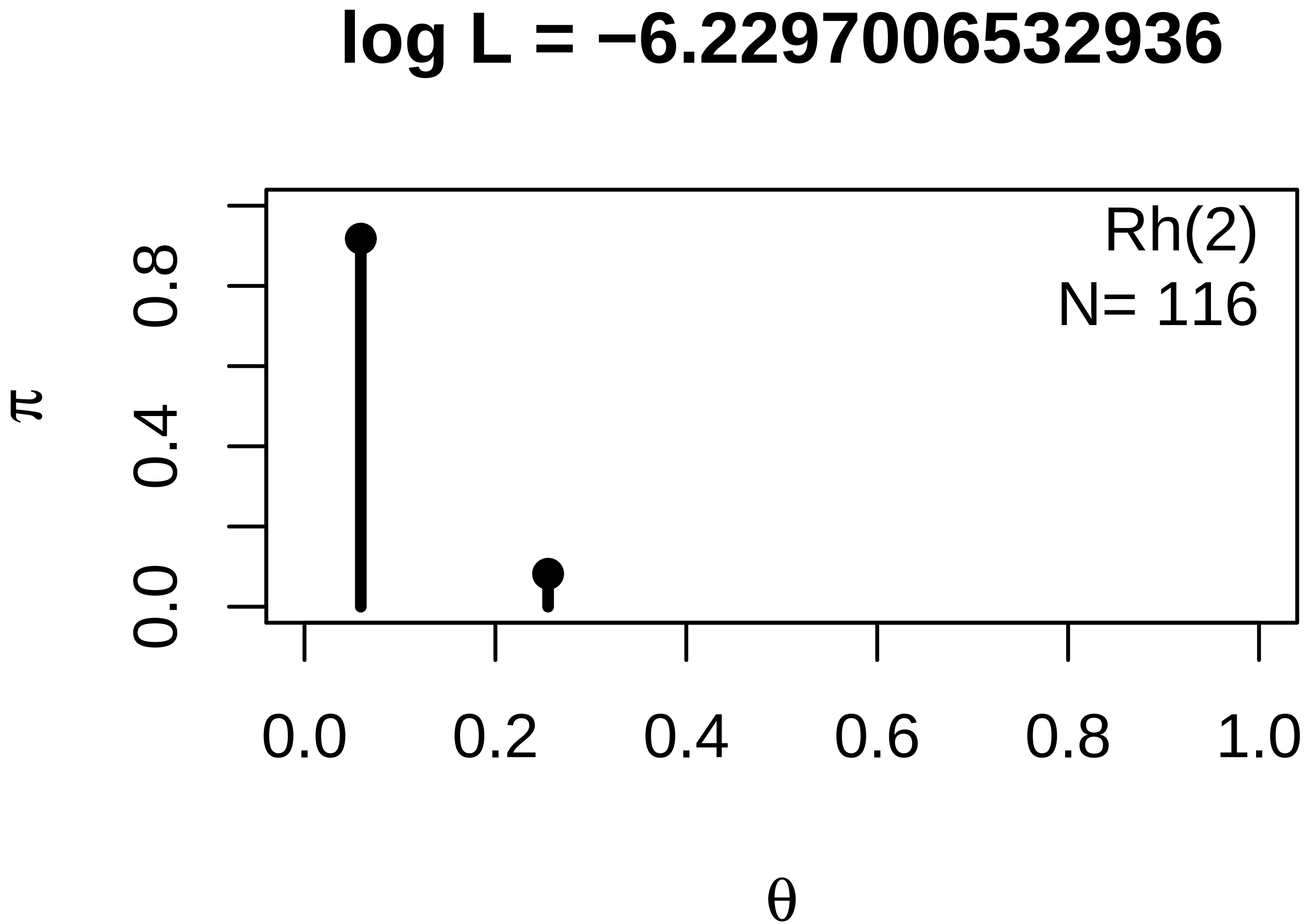

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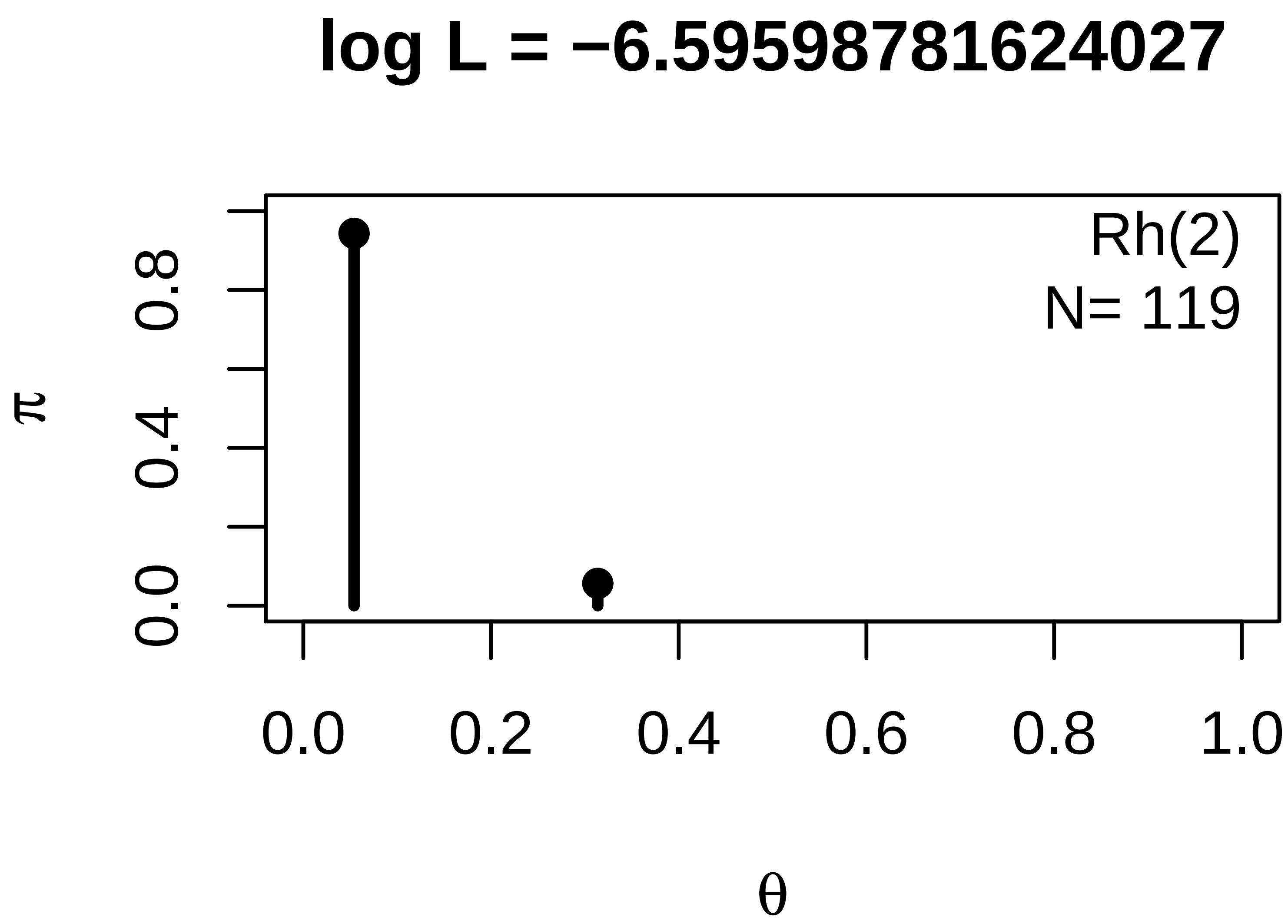

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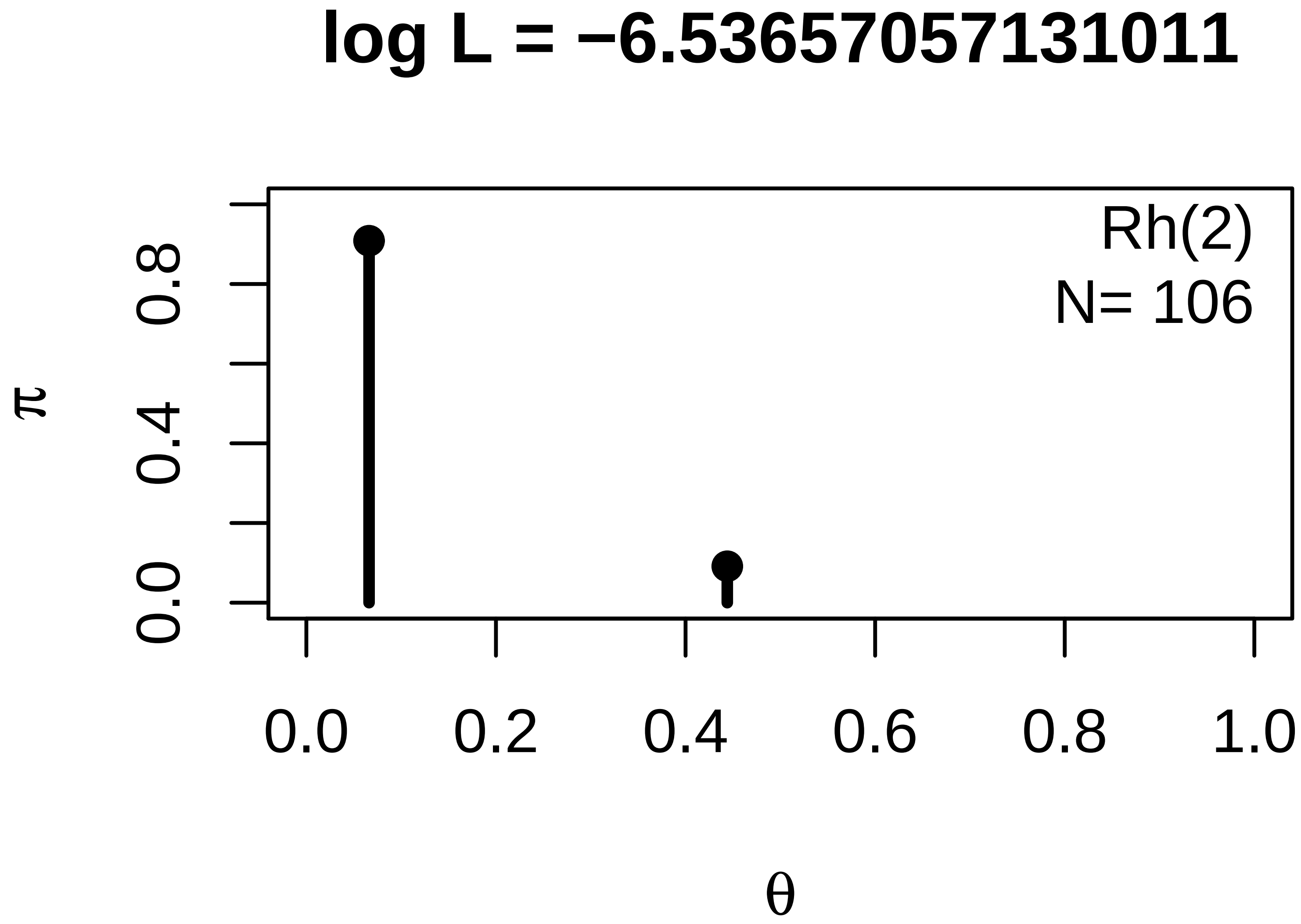

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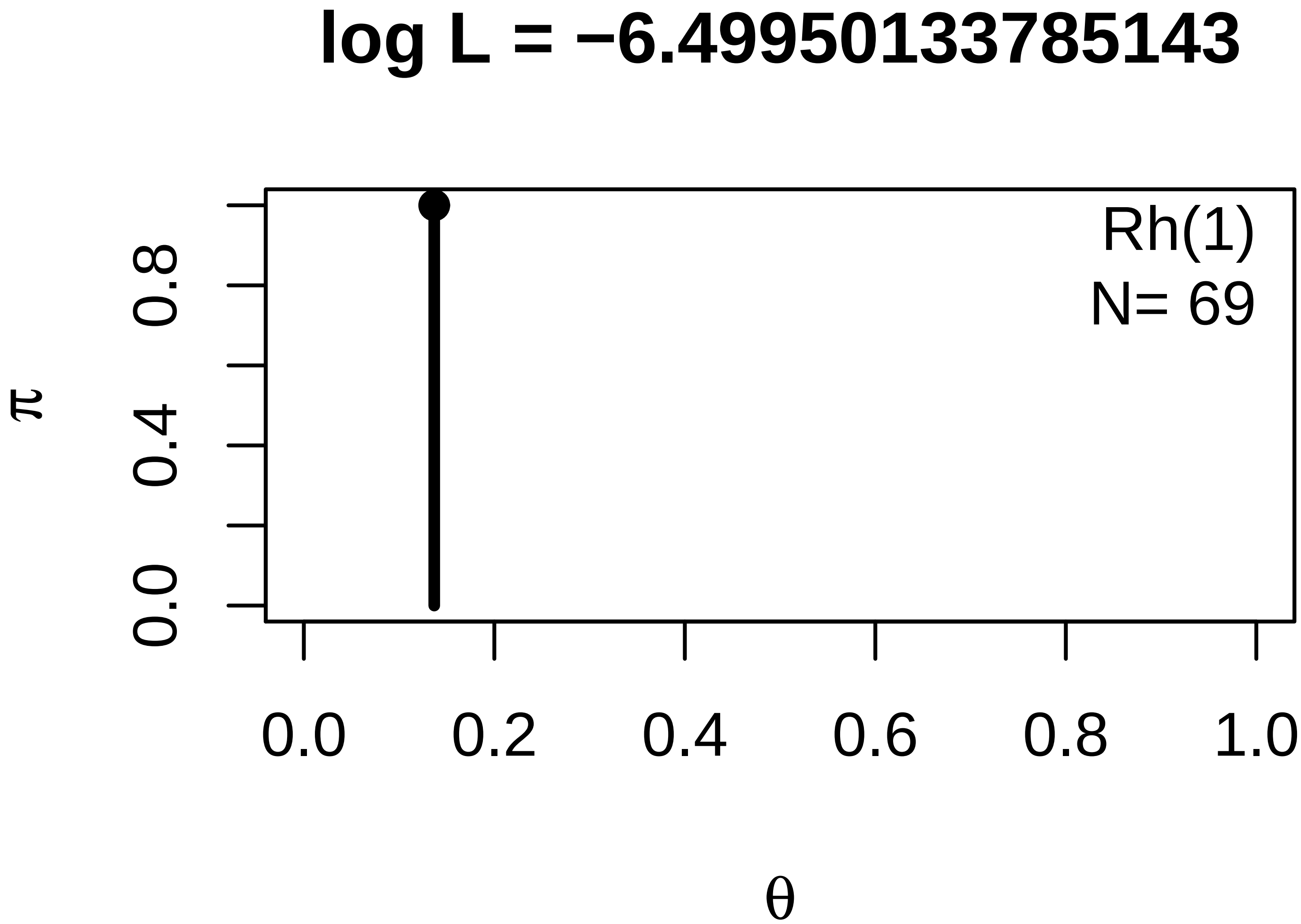

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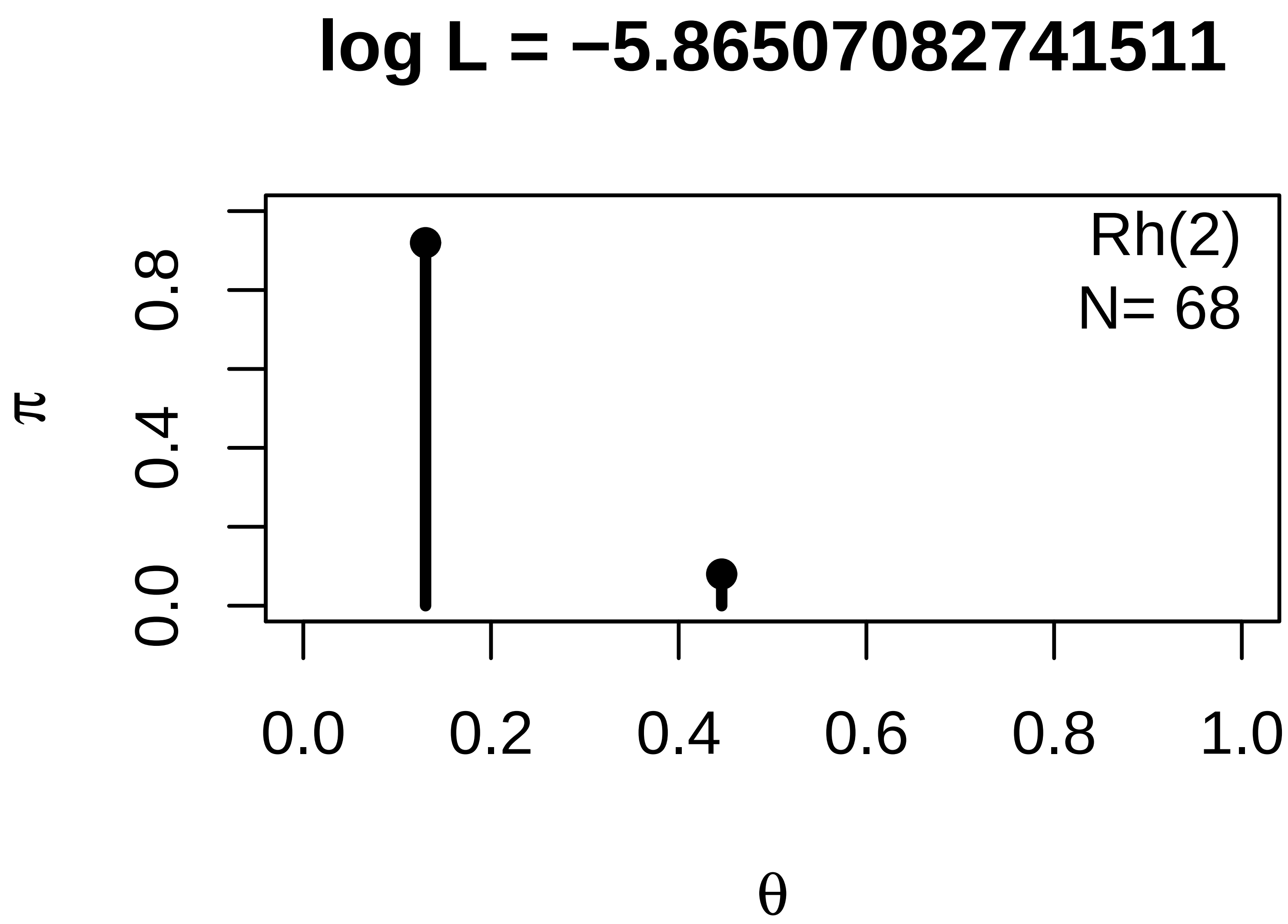

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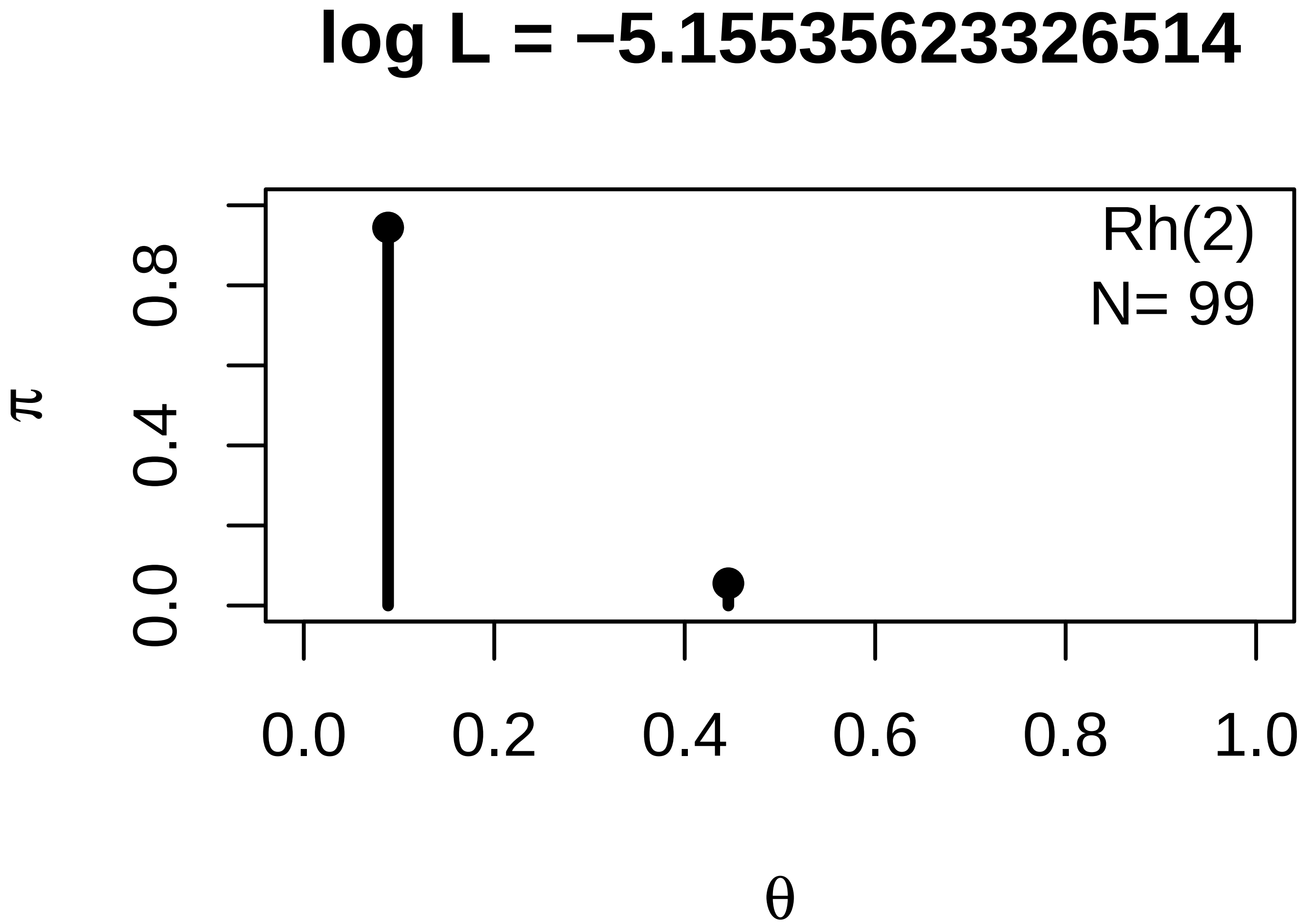

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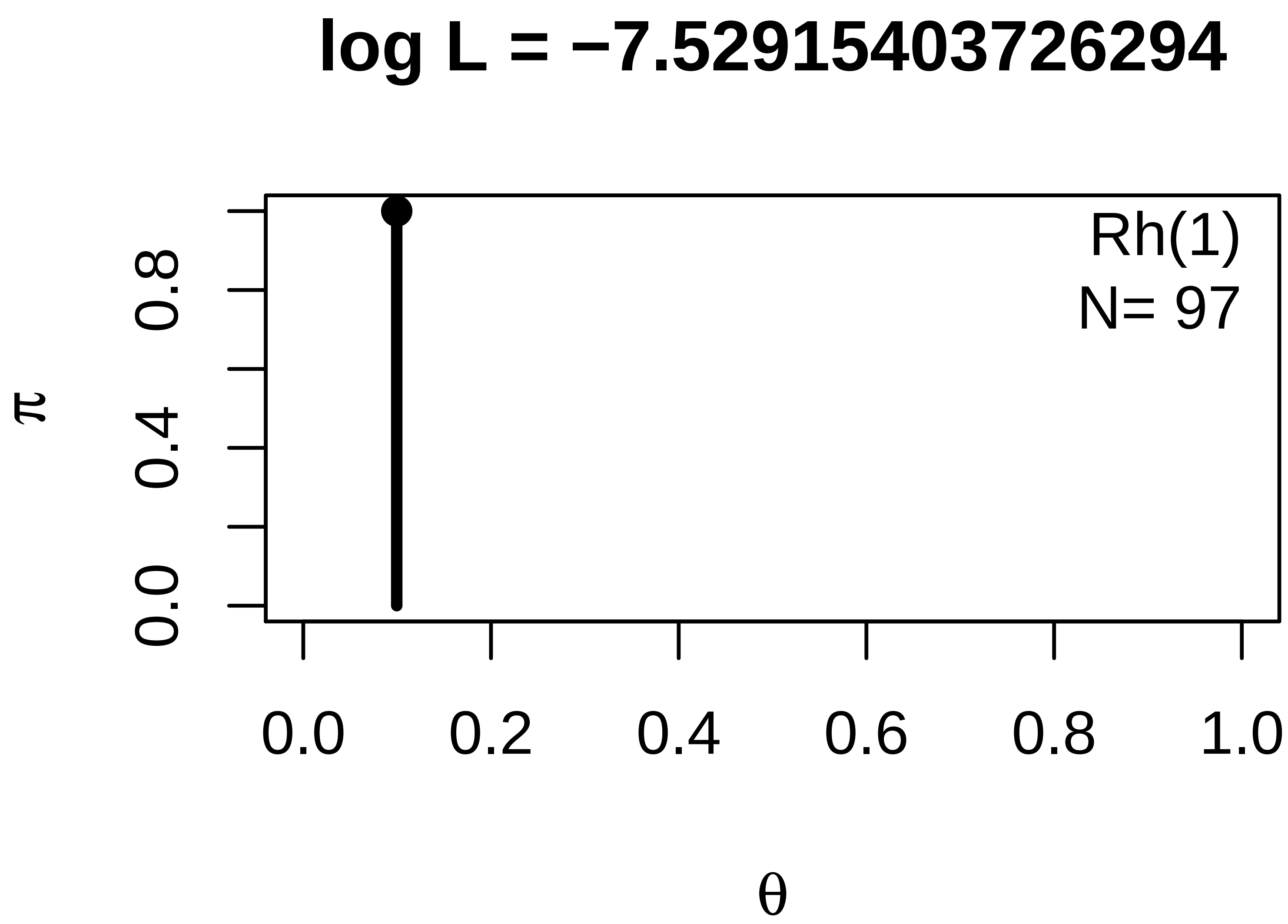

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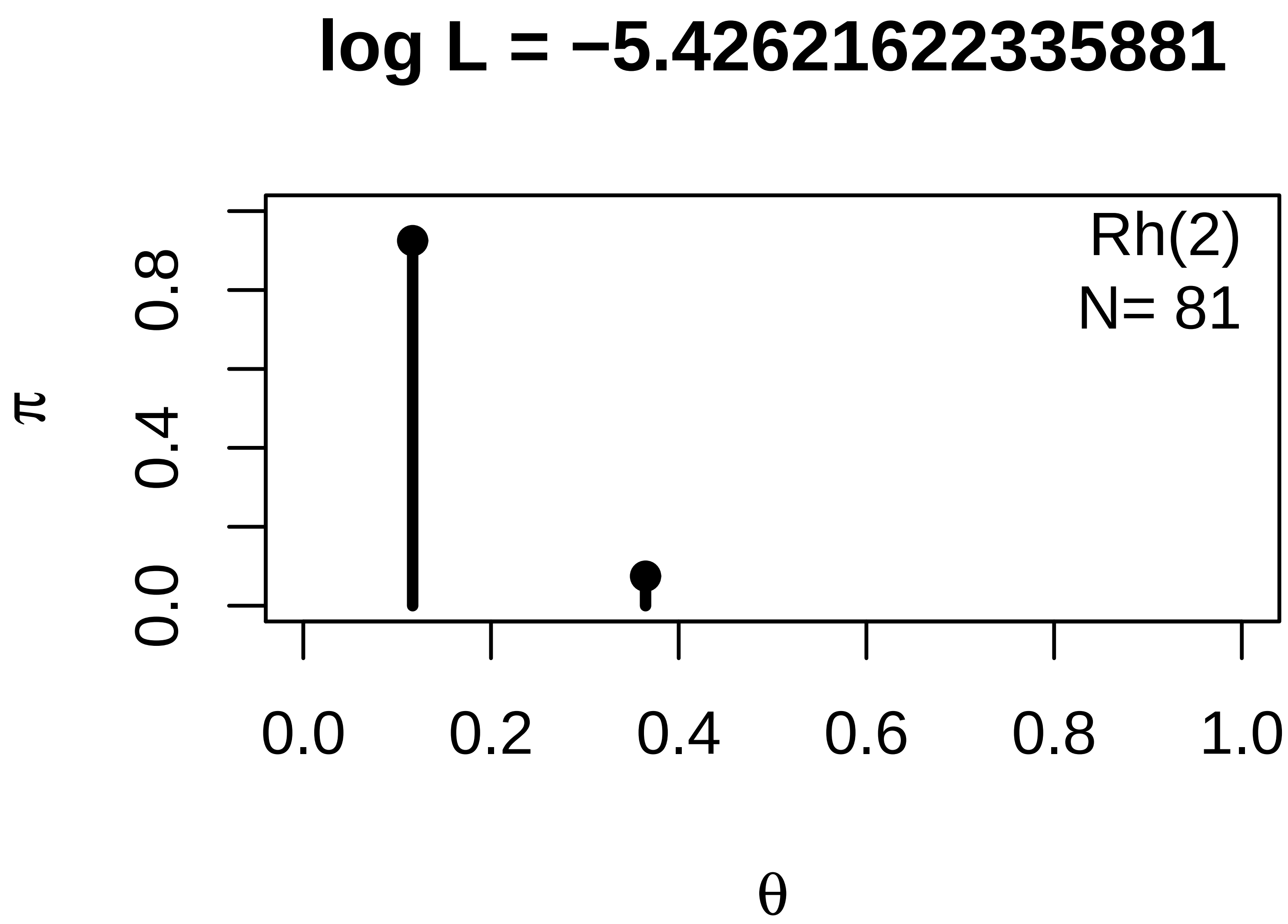

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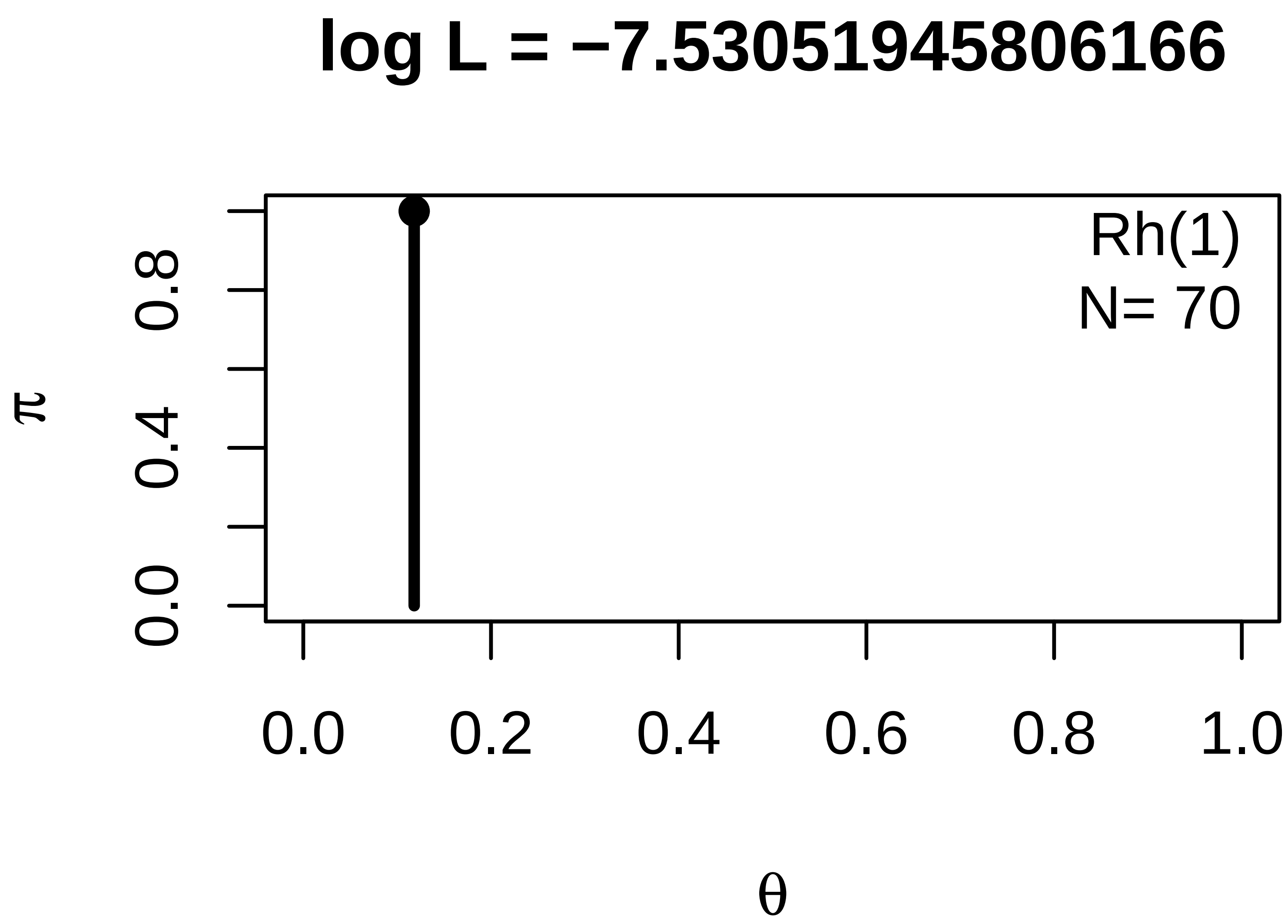

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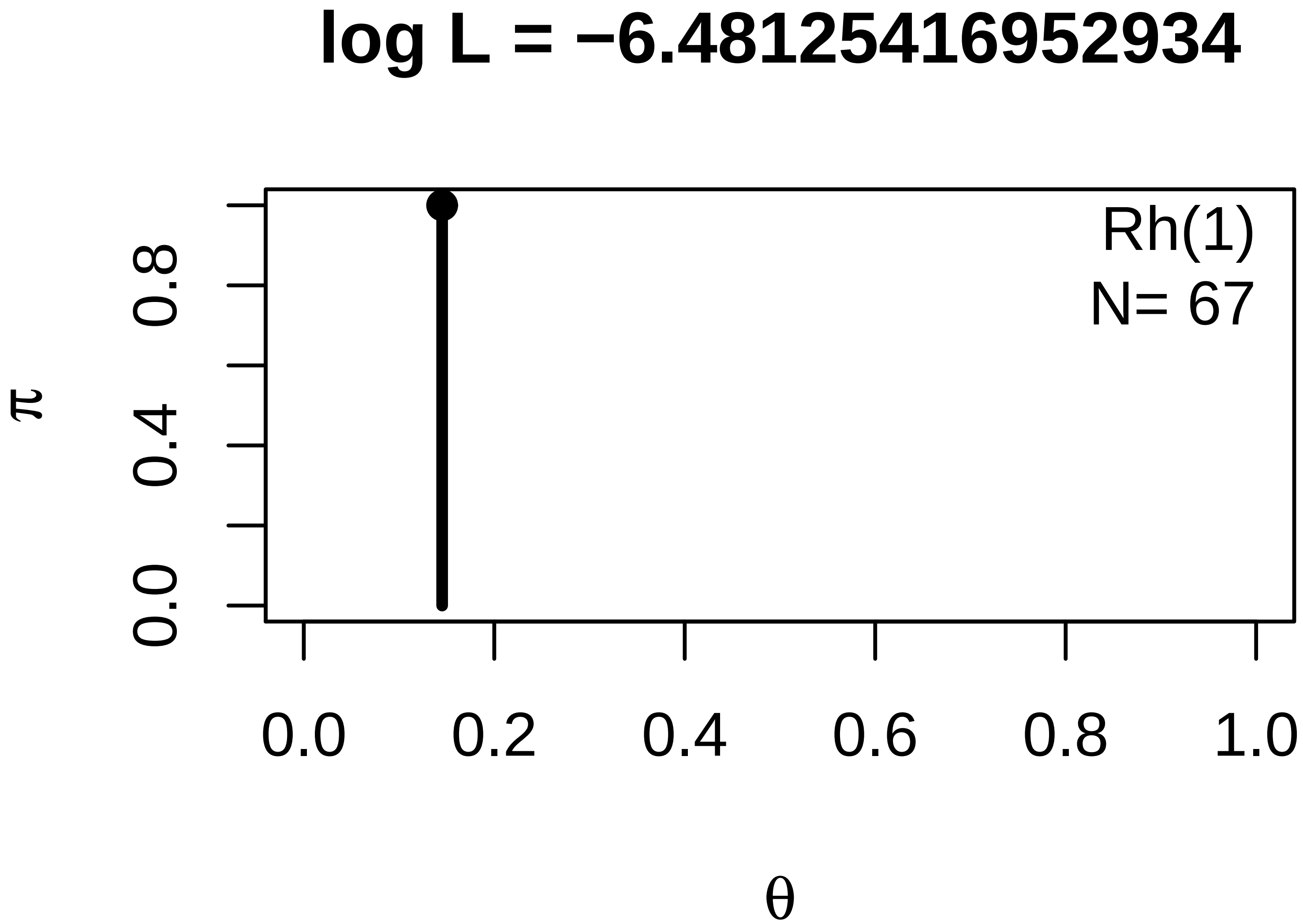

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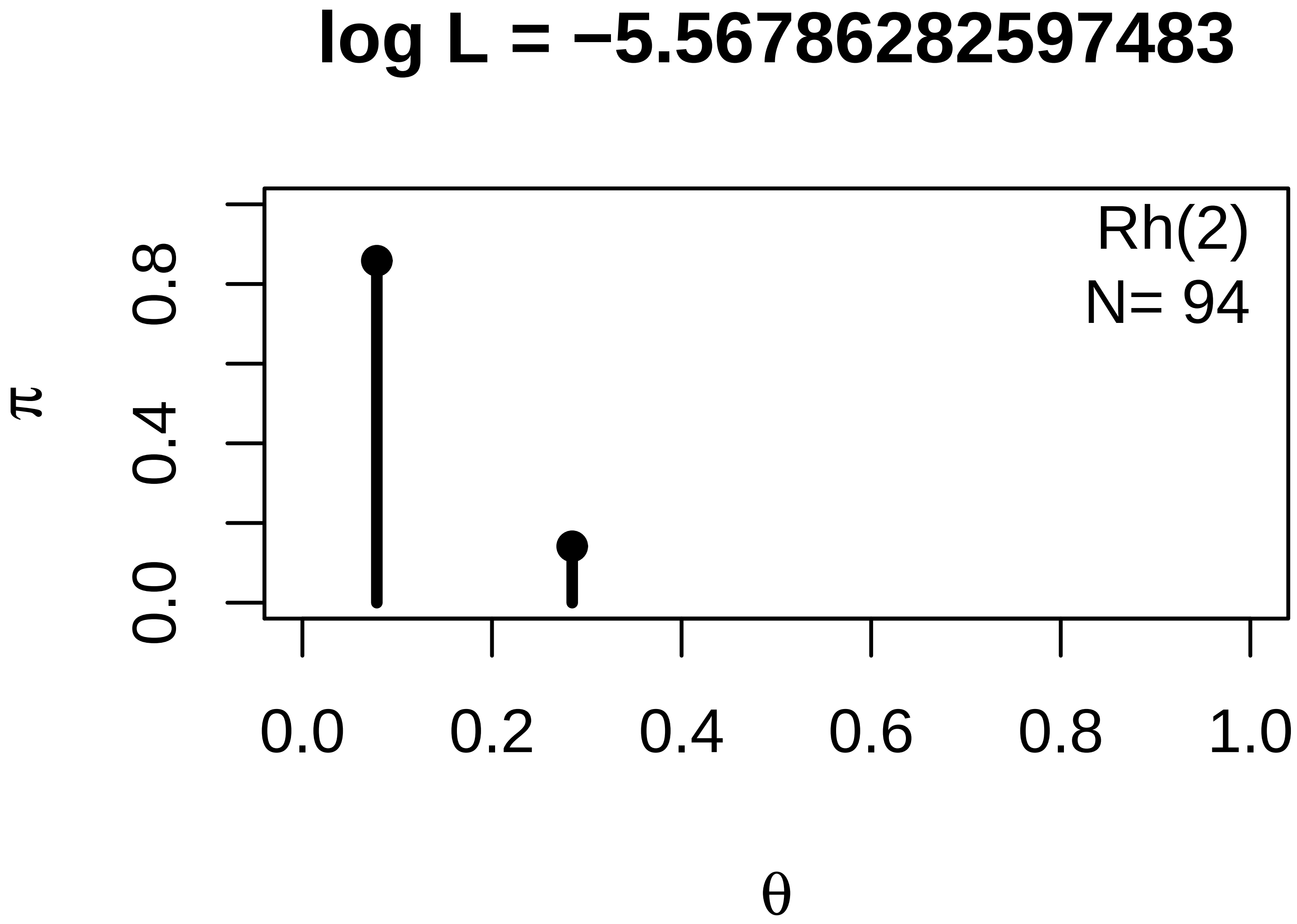

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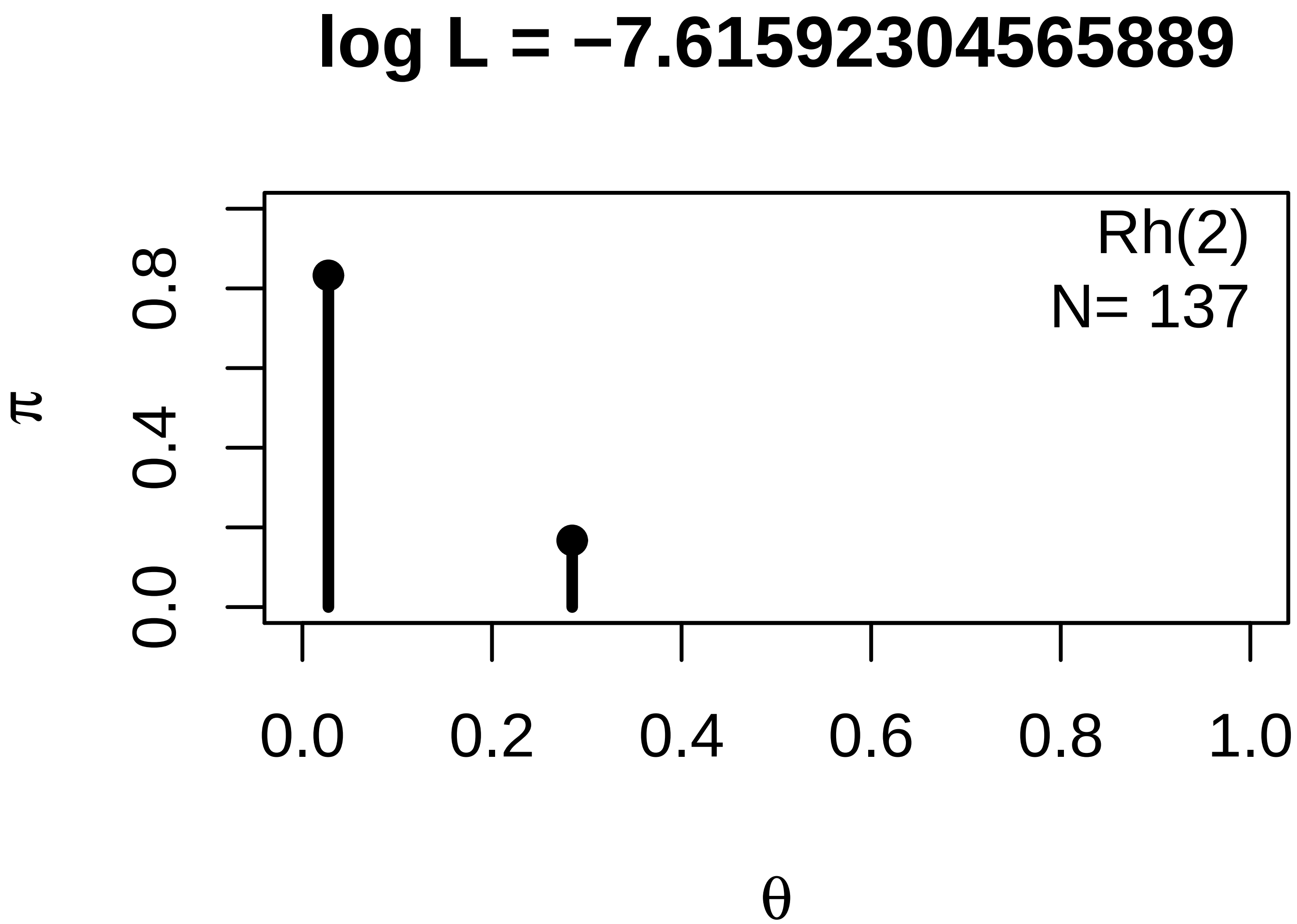

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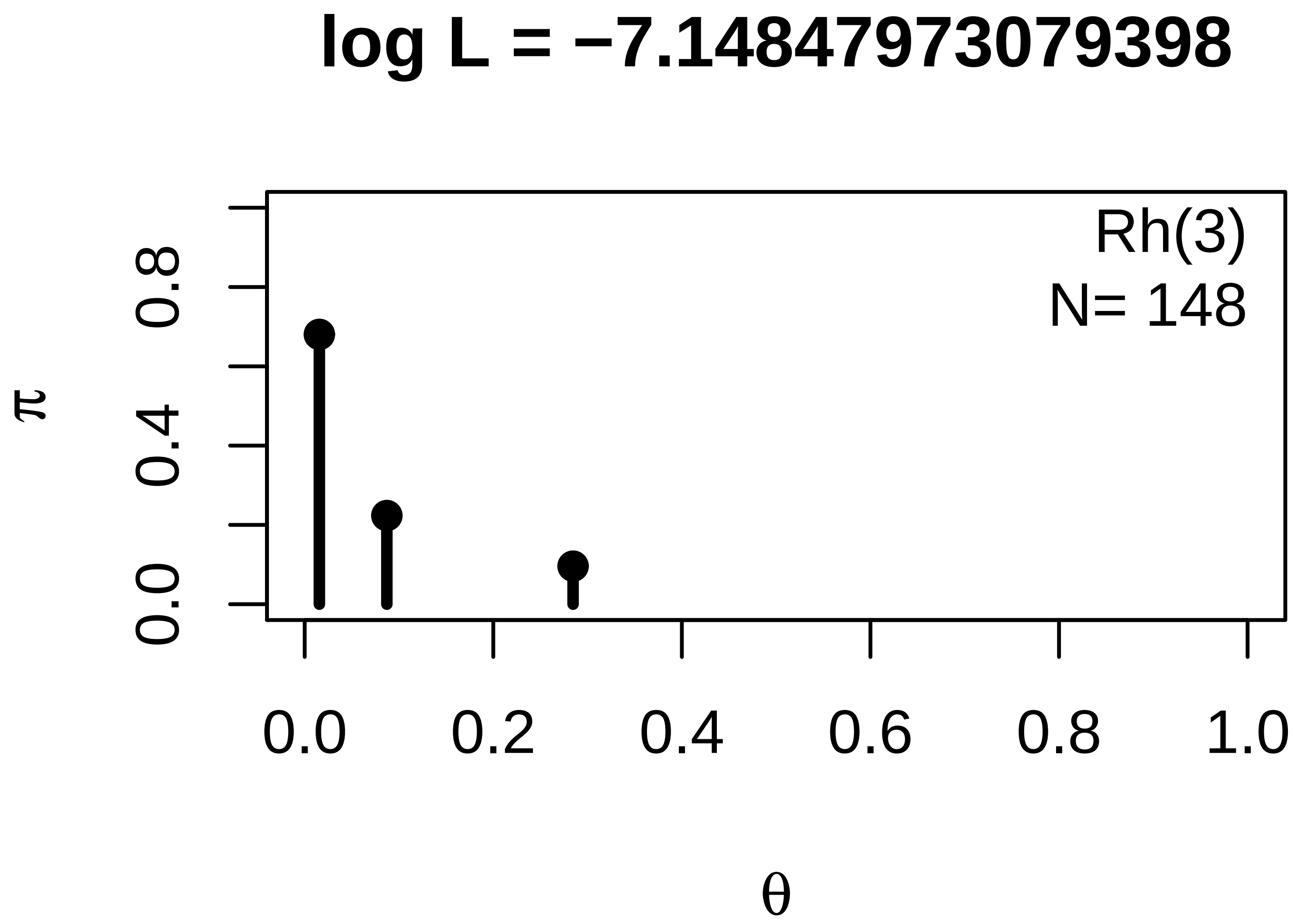

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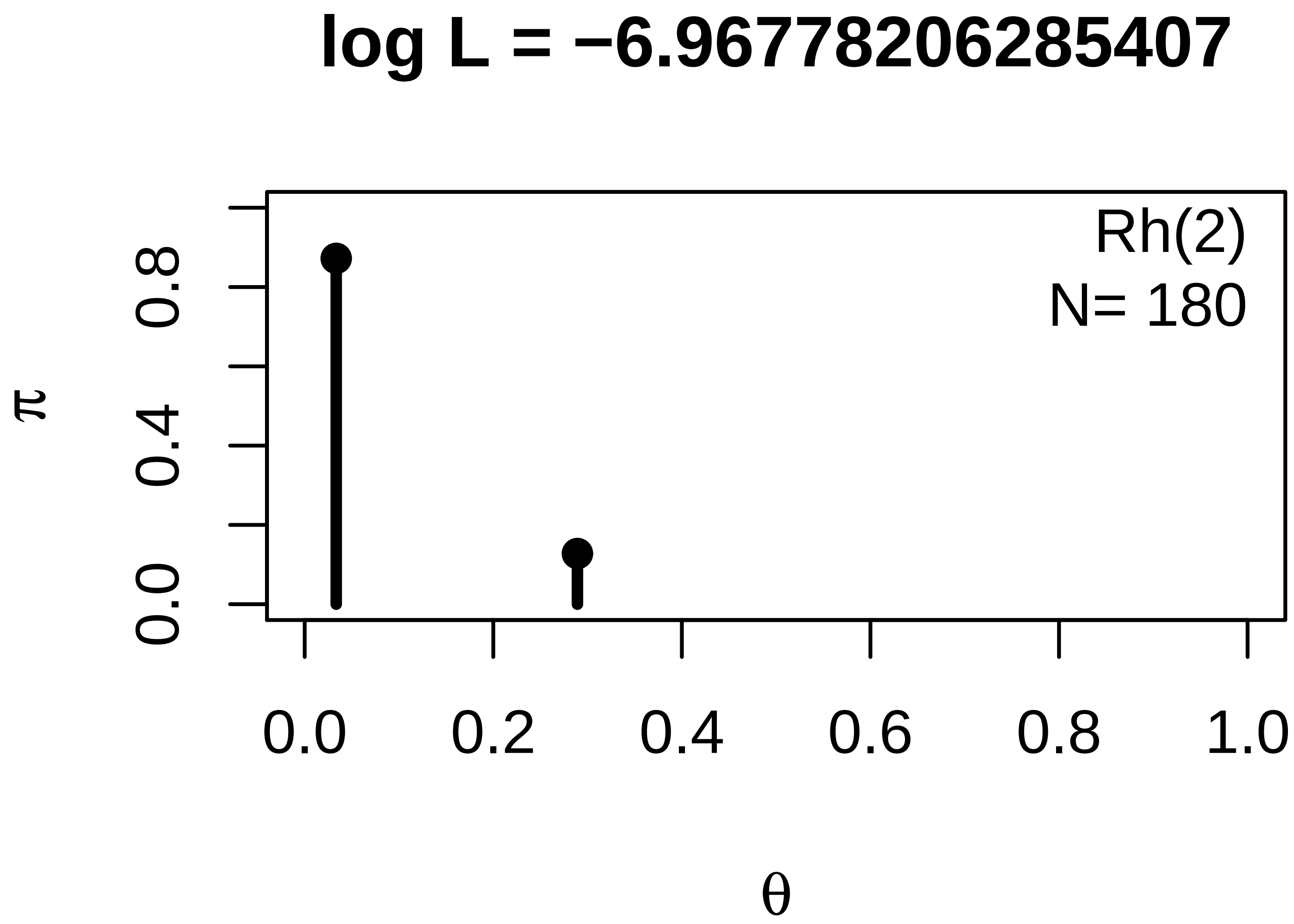
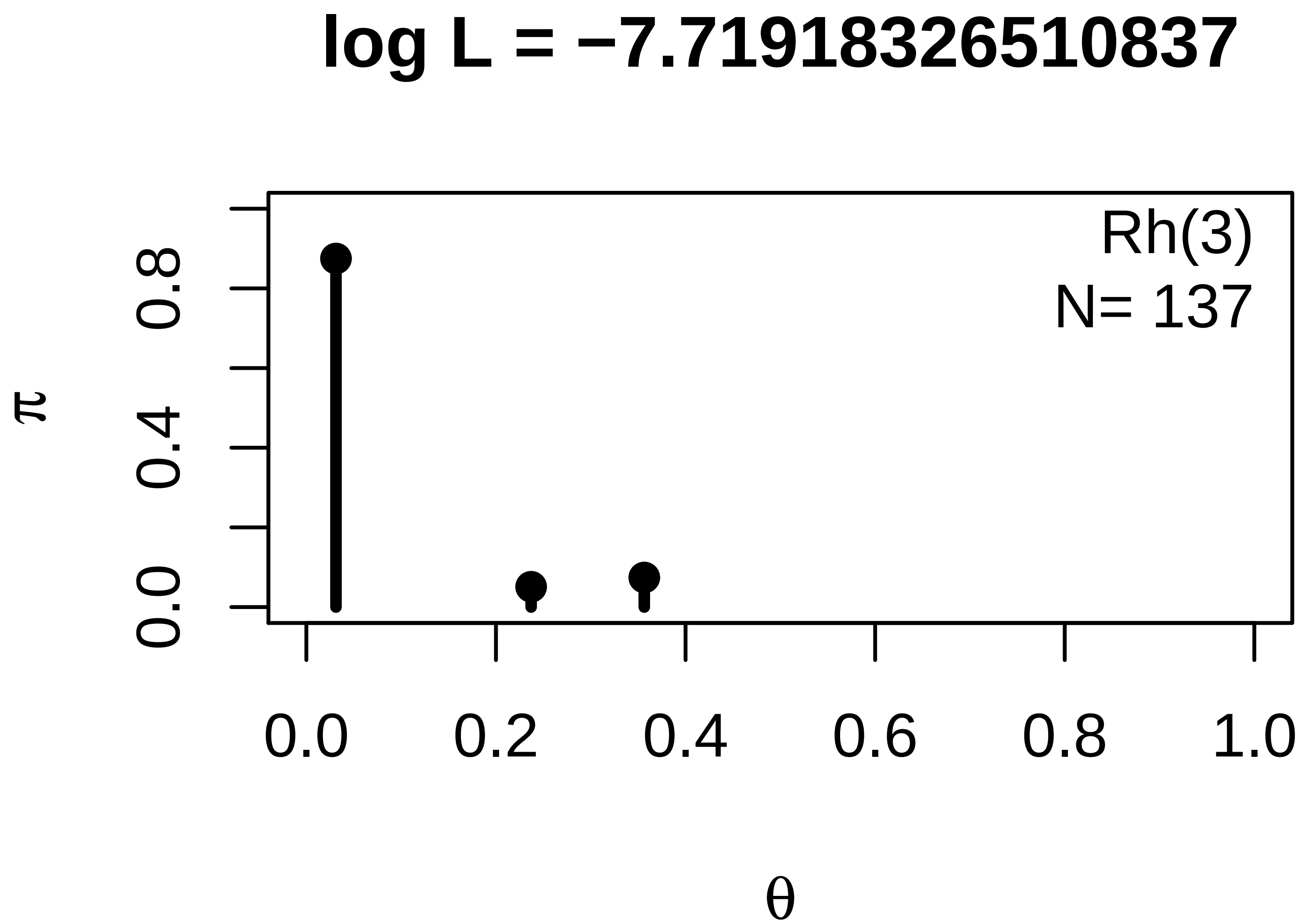
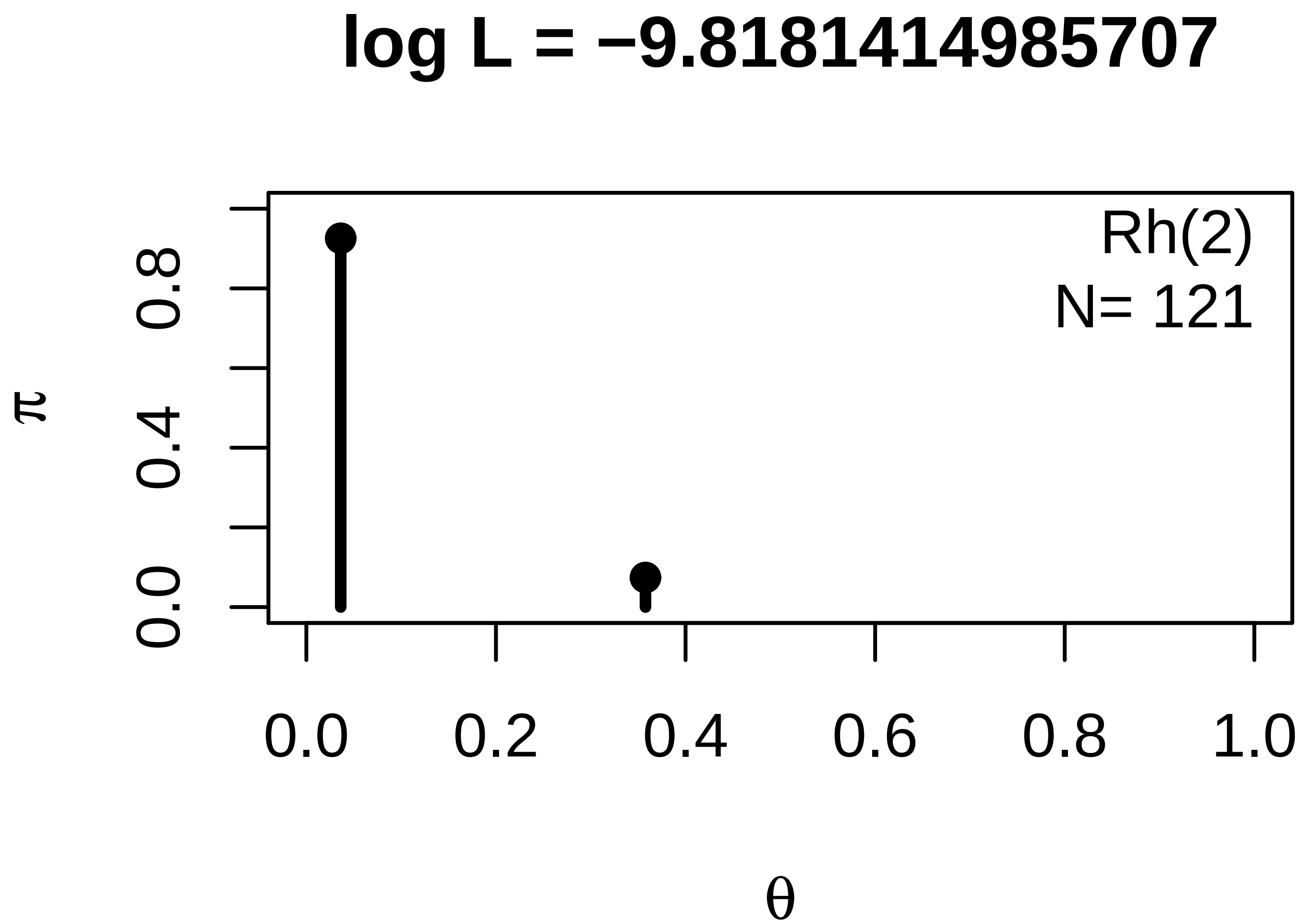


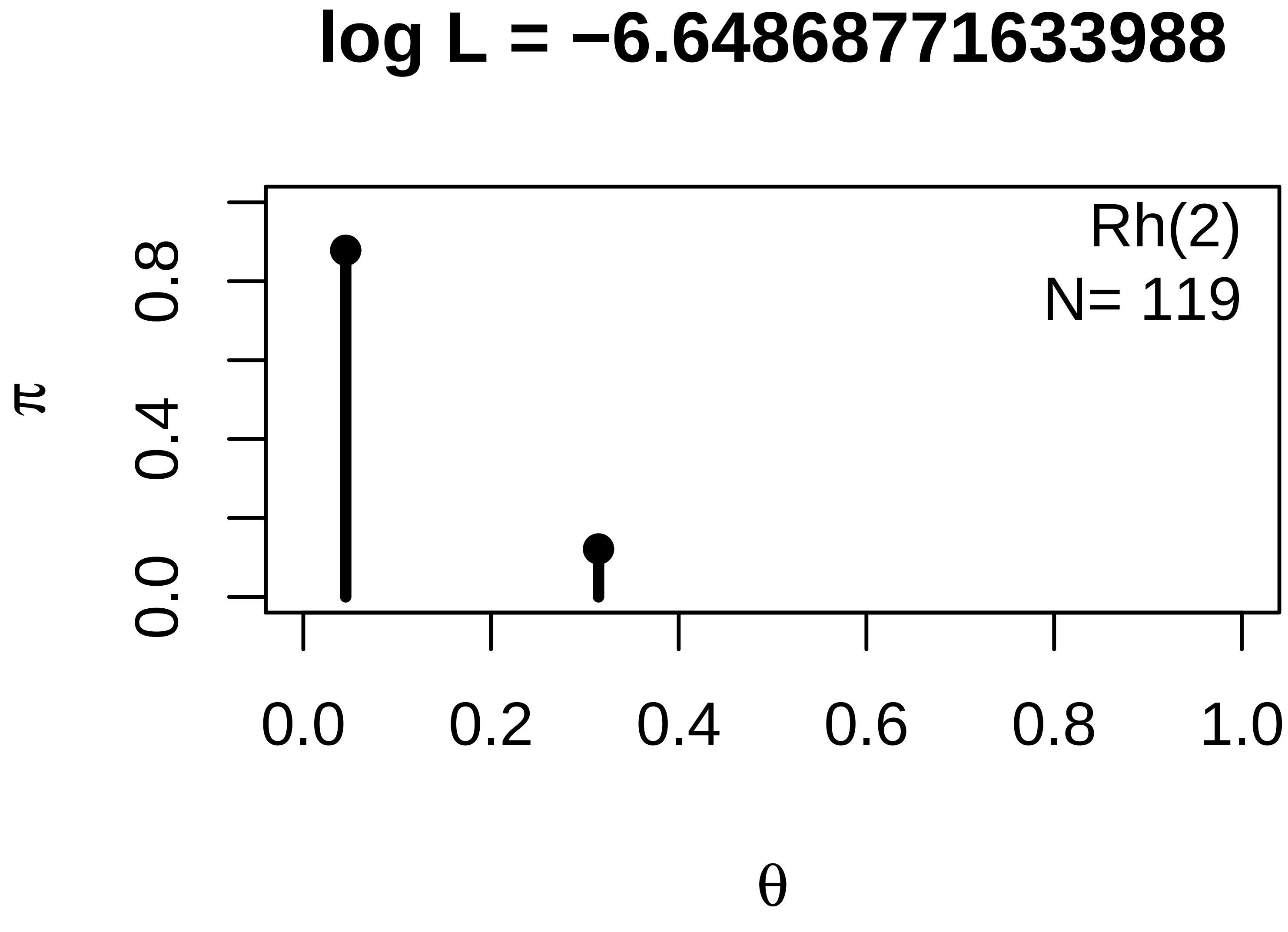
Step 97



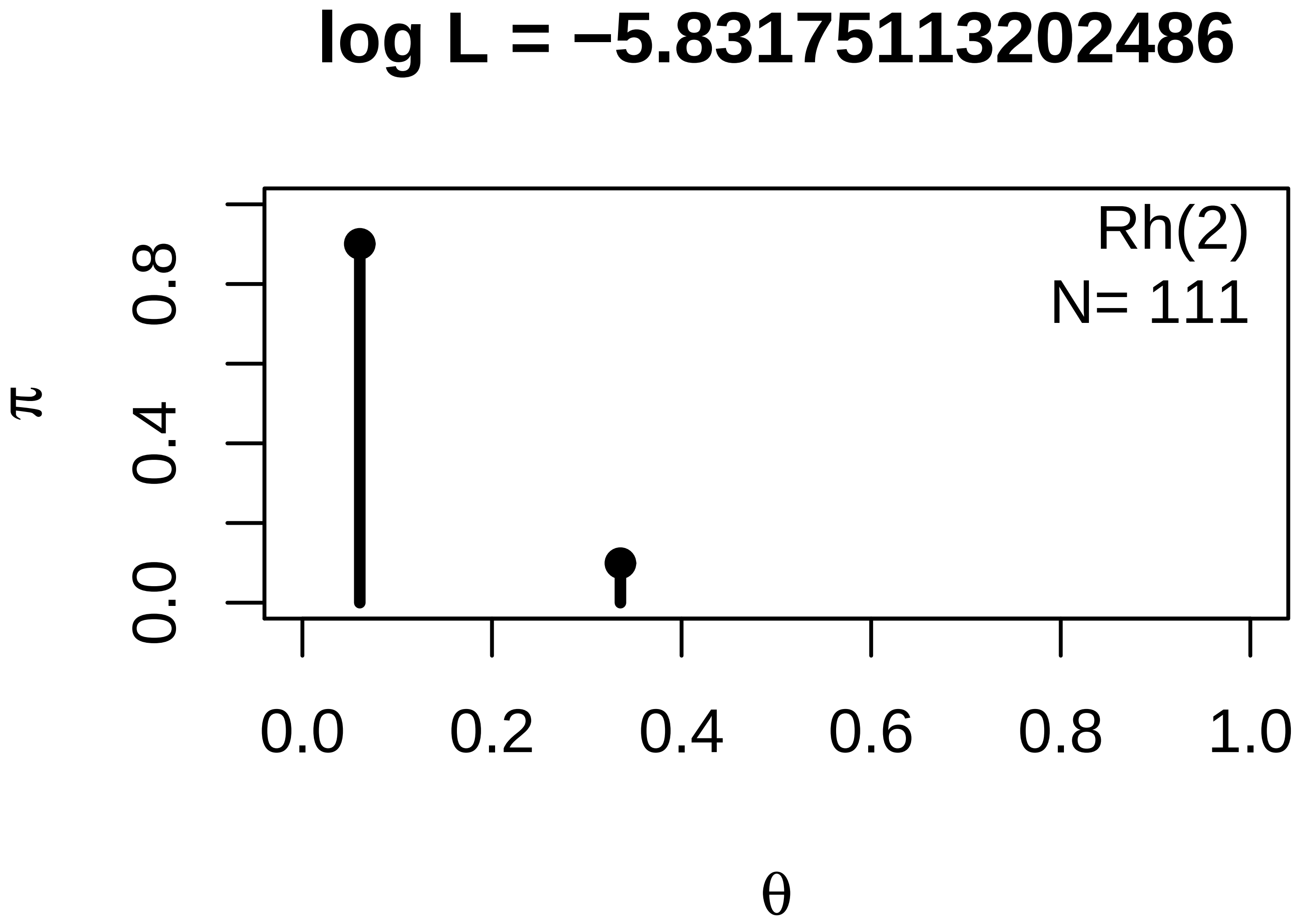
Step 98



Step 99

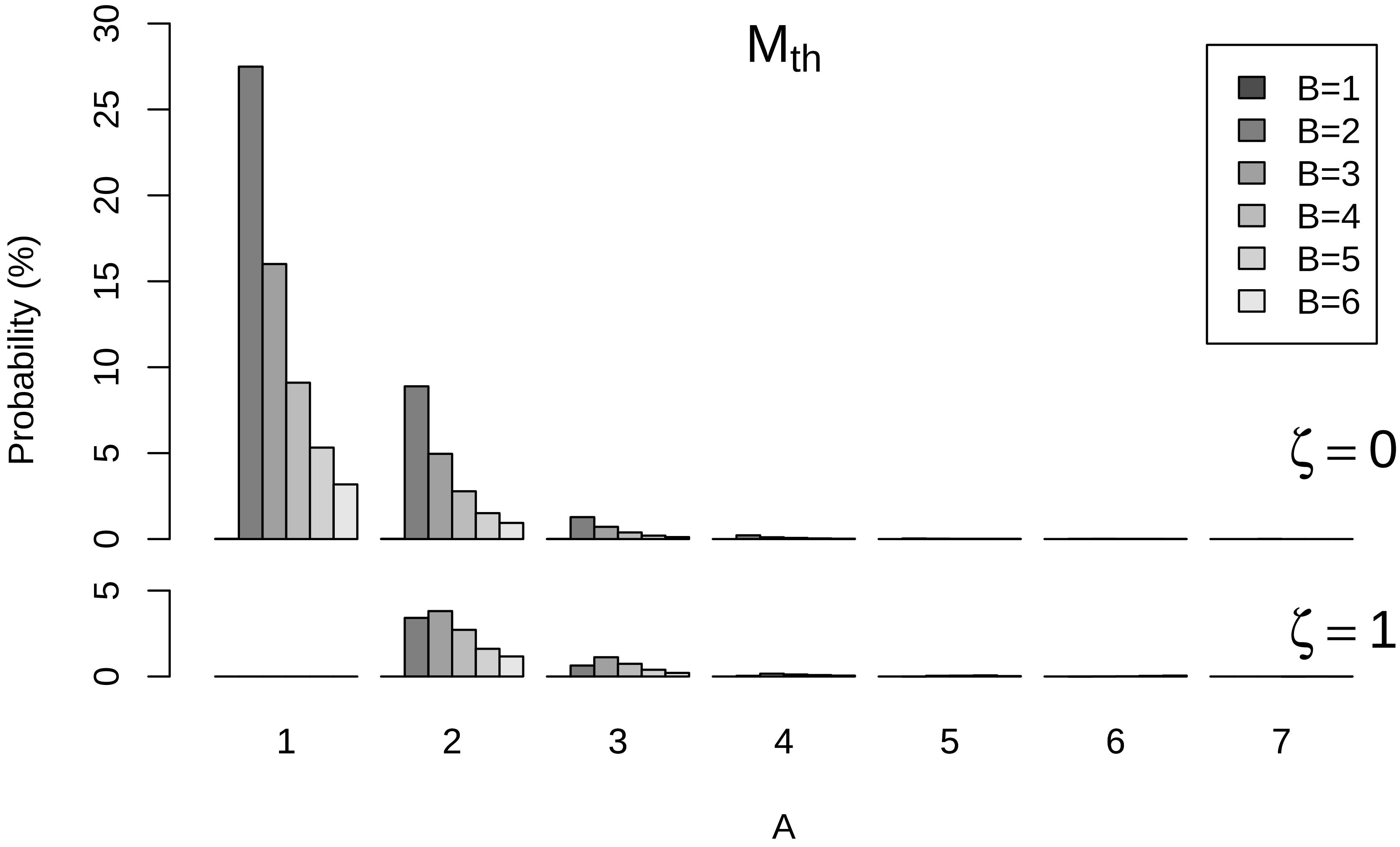


Step 100



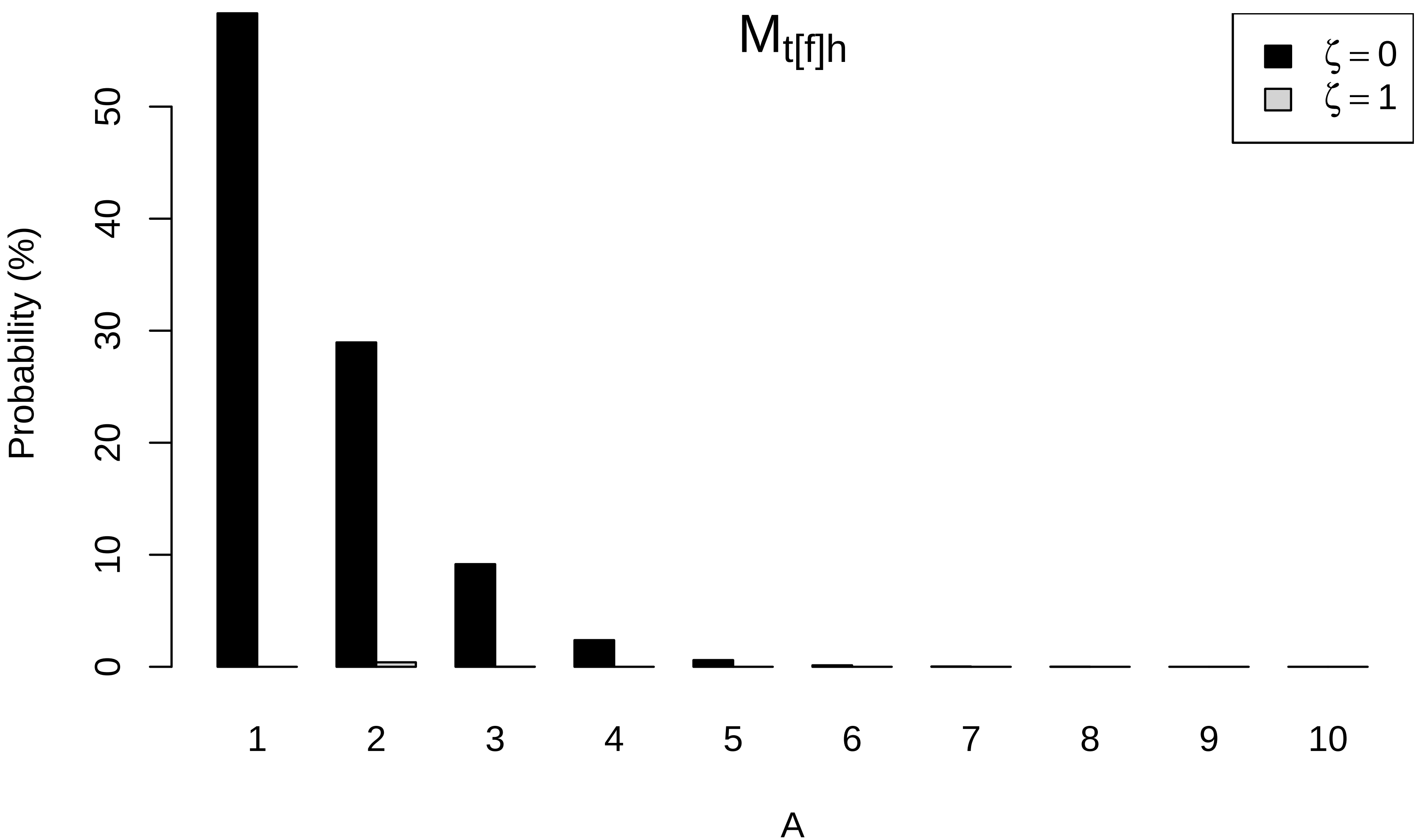
Application: AT&T Switch Testing

M_{th}: 2-way finite mixture



Application: AT&T Switch Testing

$M_{t[f]h}$: 1-way finite mixture with fixed sample effects



Posterior Model Probabilities

	$p(A,\zeta \mid X,M_{\text{t[f]h}})$		$p(A,B,\zeta \mid X,M_{\text{th}})$										
A	$\zeta=0$	$\zeta=1$	$\zeta=0$						$\zeta=1$				
$B=$			1	2	3	4	5	6	2	3	4	5	6
1	58.3			27.5	16.0	9.1	5.3	3.2					
2	29.0	0.4		8.9	5.0	2.8	1.5	0.9	3.4	3.8	2.7	1.6	1.2
3	9.2			1.3	0.7	0.4	0.2	0.1	0.6	1.1	0.7	0.4	0.2
4	2.4			0.2	0.1	0.1				0.2	0.1	0.1	0.1
5	0.6											0.1	
6	0.1												0.1
7													
8													
9													
10													

Posterior Means

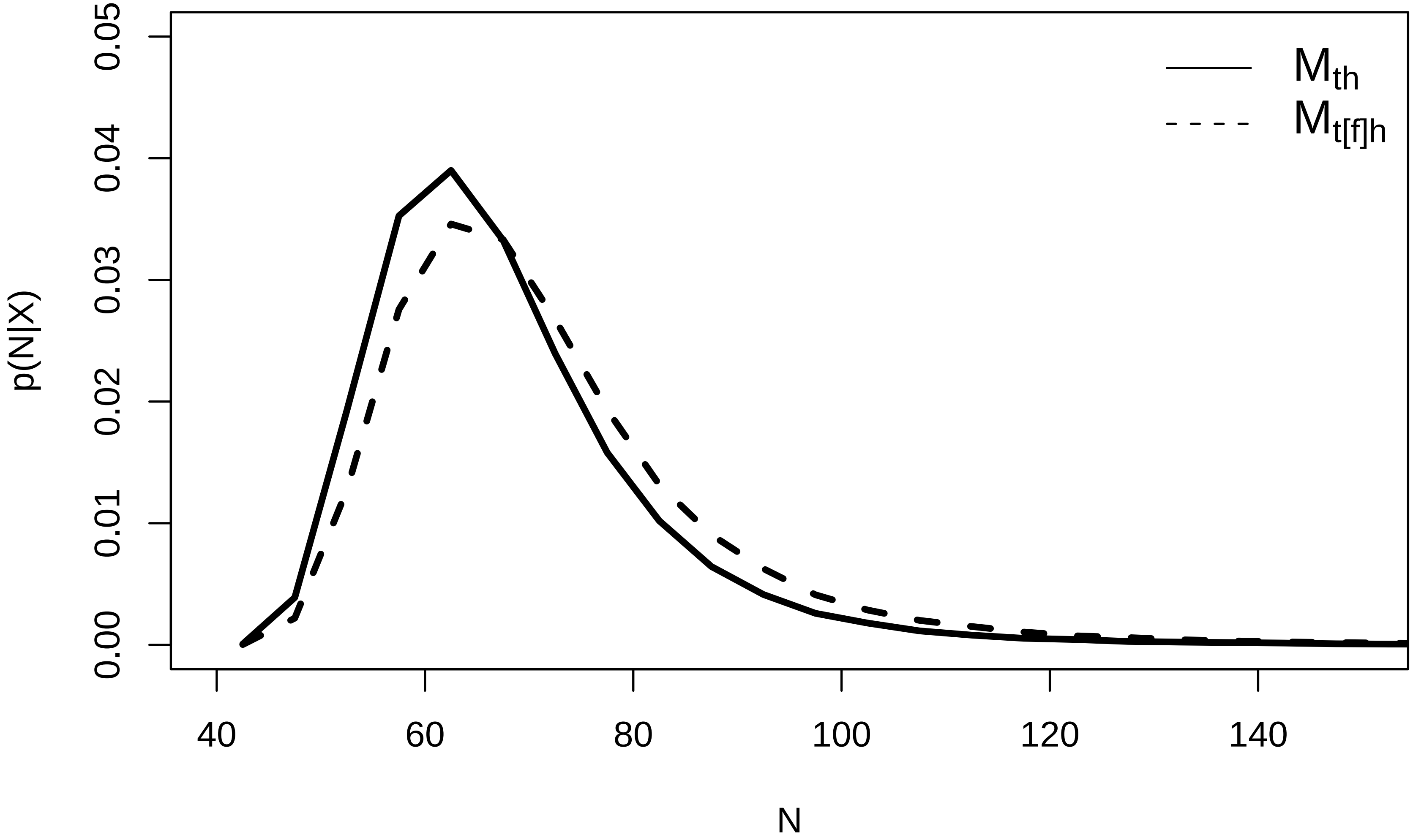
A	$E[N \mid X, M_{t[f]h}]$ $\zeta = 0$	$E[N \mid X, M_{t[f]h}]$ $\zeta = 1$	$E[N \mid X, M_{th}]$ $\zeta = 0$						$E[N \mid X, M_{th}]$ $\zeta = 1$				
1	67.8		78.5	65.9	64.7	64.3	64.2	64.2					
2	78.6	81.8	103.6	72.1	70.5	70.1	70.0	70.0	79.4	79.2	76.8	76.9	75.9
3	79.6	65.2	100.0	71.2	69.8	69.2	68.3	70.4	76.1	81.4	82.9	83.4	80.0
4	78.1			71.0	70.8	67.7	67.5	68.9	75.6	85.8	77.8	73.6	72.8
5	75.5			68.4	67.1	69.5	64.7	64.0	67.5	78.8	75.6	67.0	69.7
6	75.0			70.2	64.9	62.0	67.8	58.0	65.0	91.3	78.2	64.5	66.1
7	76.4				62.0						74.0	66.3	67.7
8	84.0												
9	83.0												
10	64.0												

Application: AT&T Switch Testing

Population size estimates $\widehat{N}$ in fixed and variable dimension models.

Model	Mean	Med.	95% CI	Npar
M_0	78.5	76	(63,101)	2
M_{t_2}	65.9	66	(54,97)	10
M_{t_3}	64.7	64	(55,77)	12
M_{t_4}	64.3	62	(60,64)	14
$M_{t_2+h_2}$	72.1	70	(63,77)	12
$M_{t_2\times h_2}$	79.4	73	(55,122)	13
M_{th}	68.7	66	(51,103)	
$M_{t[f]}$	67.8	66	(52,94)	8
$M_{t[f]+h_2}$	78.6	74	(54,129)	10
$M_{t[f]\times h_2}$				15
$M_{t[f]h}$	72.4	69	(52,114)	

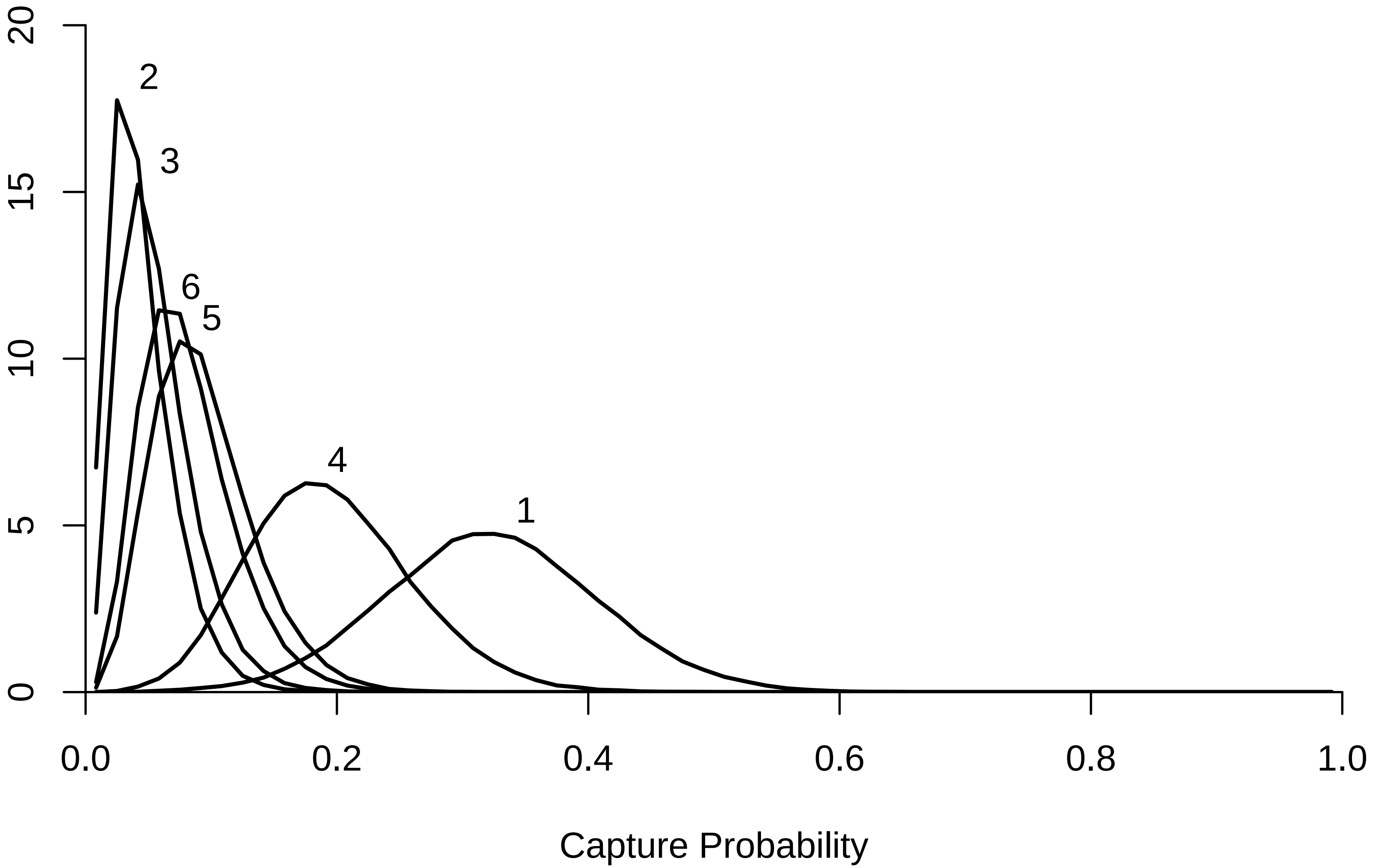
Posterior distribution of N



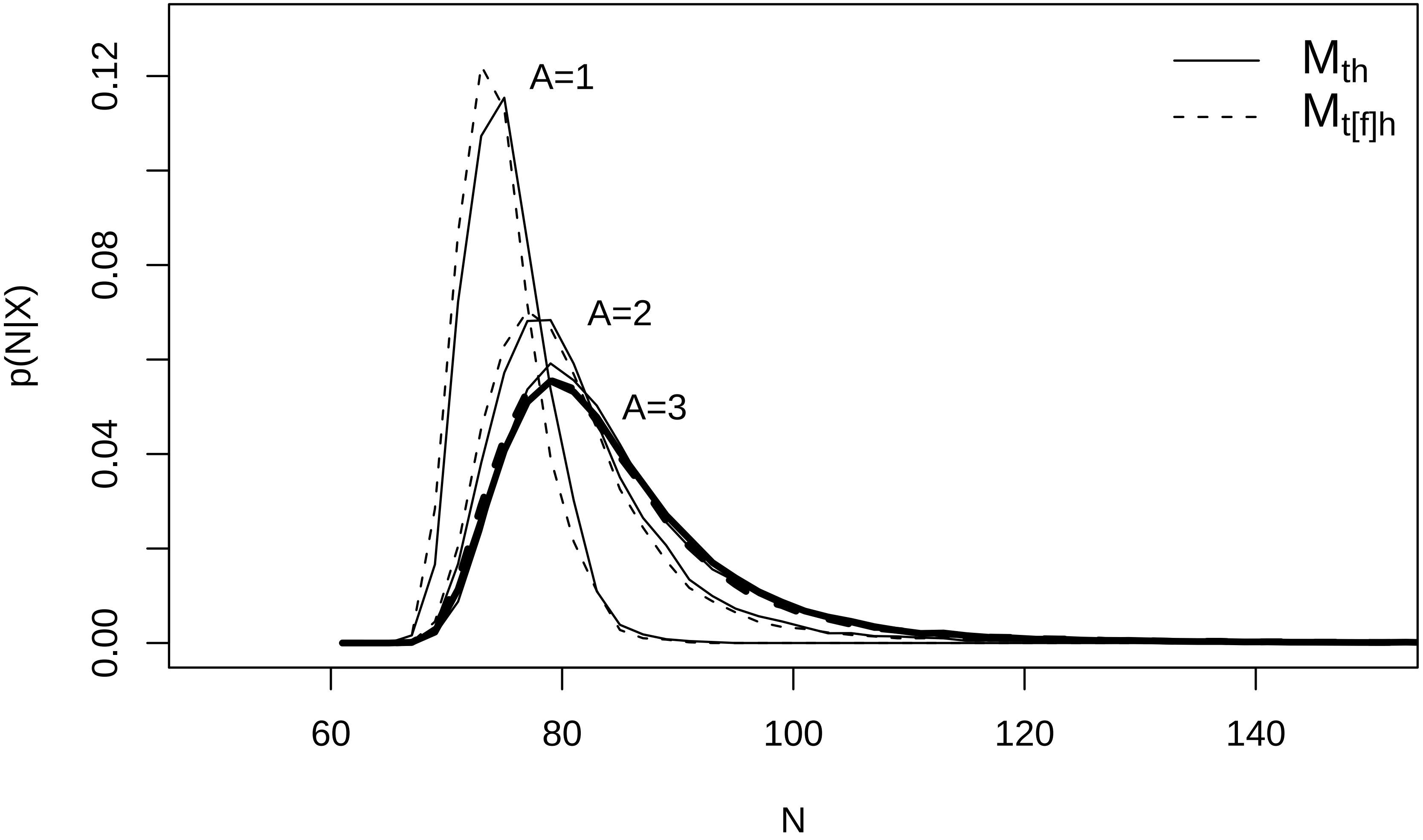
Application: AT&T Switch Testing

- Models M_{th} and $M_{\mathrm{t[f]h}}$ both predict of the order 70 errors in total (95% credible intervals [51,103] and [52,114])
- $\sim$ 30 errors not so far detected
- Evidence for heterogeneity among **reviewers**, but not amongst faults
- Bayes Factor $\mathrm{BF}(M_{\mathrm{th}} : M_{\mathrm{t[f]h}}) = 0.42$: neither model class convincingly favoured.

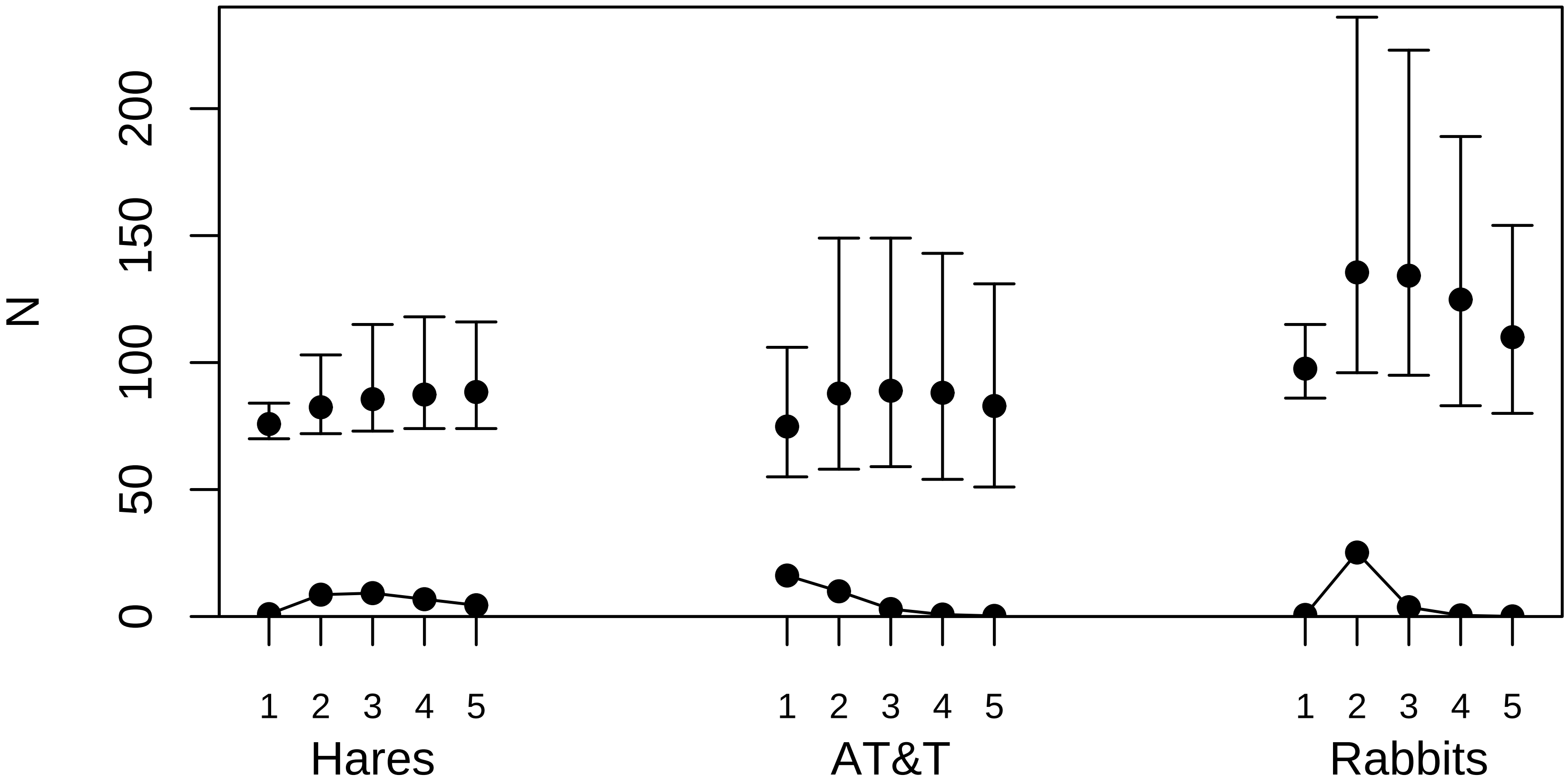
Skills of the 6 reviewers



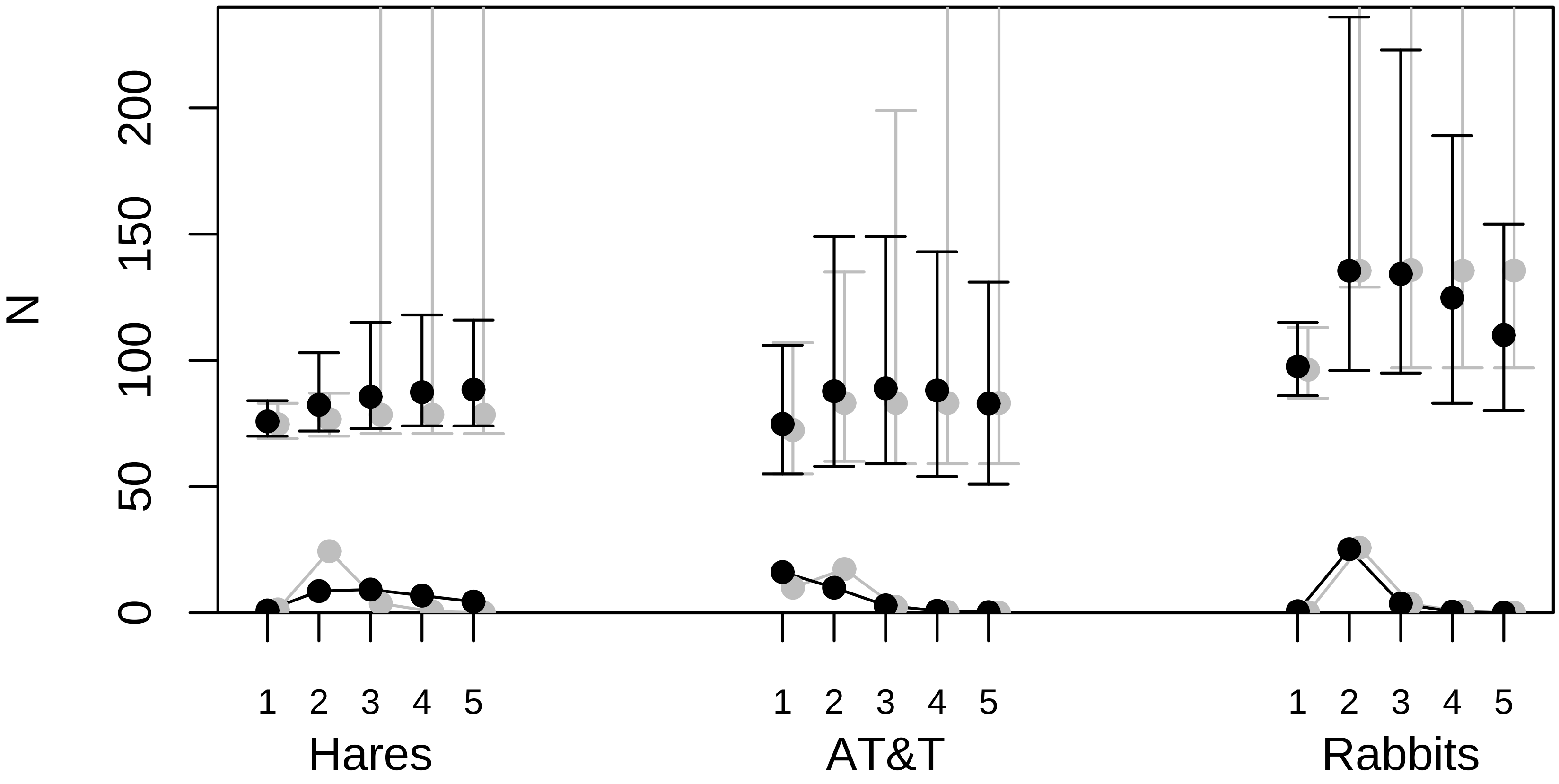
Application: Snowshoe Hares



Interval estimates – 3 examples



Interval estimates – 3 examples



Summary

- RJMCMC is a practicable means for model selection/averaging in Capture-Recapture with finite mixtures.
- Doesn't solve non-identifiability between model classes
- Priors regularise the likelihood – adding extra components does not affect estimates much.