The `R` package `stelfi`

Charlotte M. Jones-Todd, University of Auckland

Biometrics in the Bay of Islands, 2023

`work with: Xiangjie Xue`

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Model Buil**

Charlotte M. Jones

The R package stelfi

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work with: Alec va

stelfi

CRAN: R package version 1.0.1. (2023)

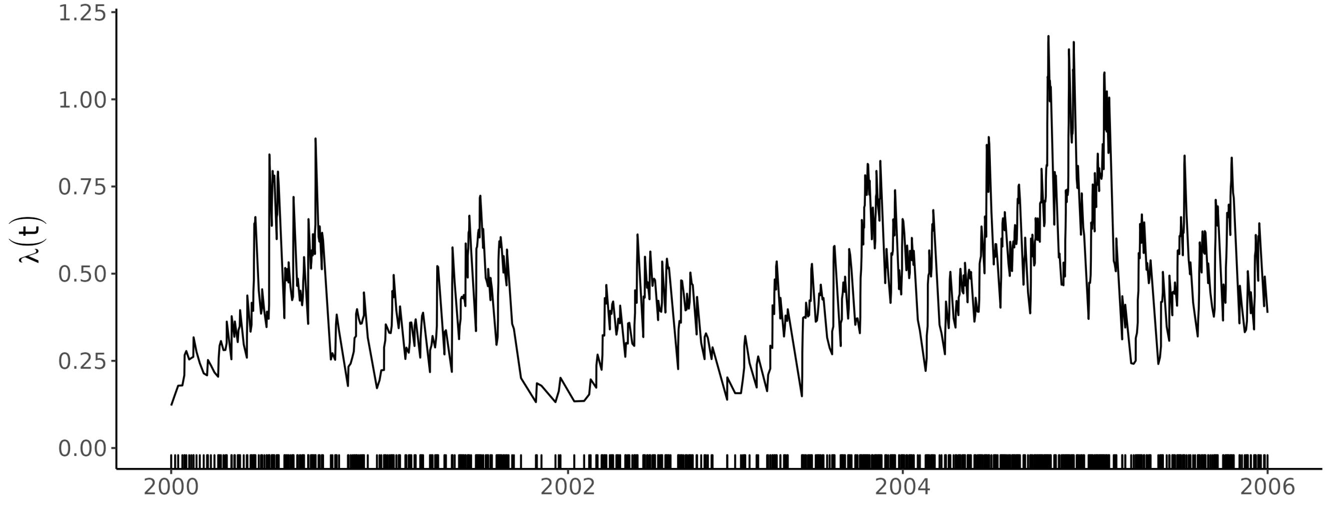
stelfi: Hawkes and Log-Gaussian Cox Point Processes Using Template Model Builder

Charlotte M. Jones-Todd; Alec van Helsdingen

`TMB`: `R` pa

- Calcu
  object
- User s

Models in
`C++` templ



`TMB`: `R` package for fitting latent variable models

- Calculates first and second order derivatives of the likelihood function by AD (or any objective function written in `C++`)
- User specifies which function arguments the Laplace approximation should be applied to

Models in `stelfi` are fitted via maximum likelihood using `TMB` via a range of custom written `C++` templates

Spatial LGCP

Spatial

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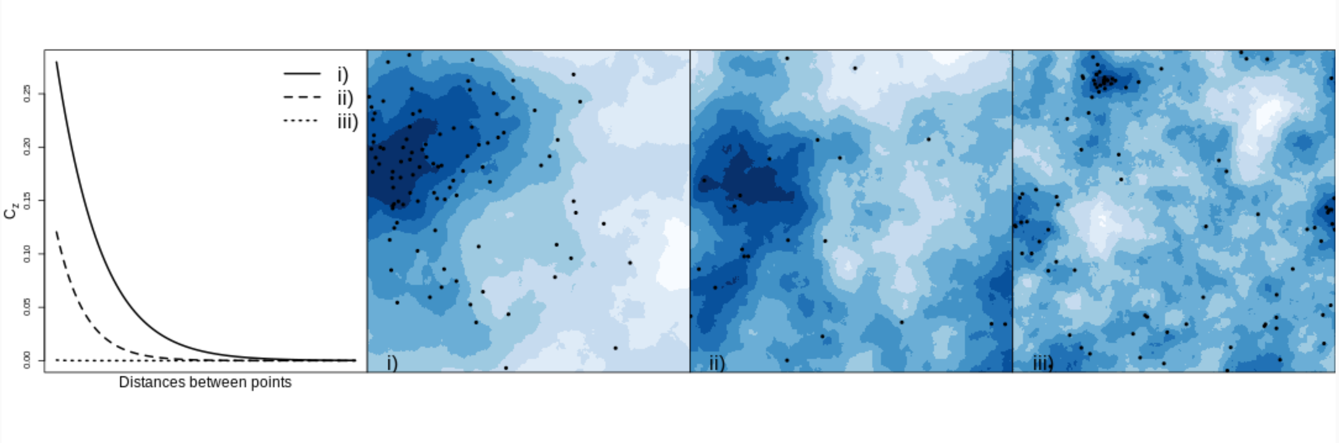
`fit_lgcp(`

- `locs`,
- `sf`, ar
- `smesh`
- `param`



Spatial LGCP

Spatio

$$\Lambd

- \(\bet
- \(G(\u
- \(\eps

fit_lgcp(

- locs ,
- sf , ar
- smesh
- tmesh
- param

$$\Lambda(\underline{s}) = \exp(\beta_0 + G(\underline{s}) + \epsilon)$$

- β_0, intercept
- $G(\underline{s})$, Gaussian Markov Random Field (GMRF)
- ϵ, error

$G(\underline{s})$ has a Matérn covariance defined by τ and κ, where

- $r = \frac{\sqrt{8}}{\kappa}$ is the range, and
- $\sigma = \frac{1}{\sqrt{4\pi\kappa^2\tau^2}}$ standard deviation

```
fit_lgcp(locs, sf, smesh, parameters)
```

- `locs`, matrix of locations
- `sf`, an `sf` object of the spatial domain
- `smesh`, a `INLA::inla.mesh.2d` object
- `parameters`, vector of parameter starting values

Spatiotemporal LGCP

A mark

$$\Lambda(\underline{s}, t) = \exp(\beta_0 + G(\underline{s}, t) + \epsilon)$$

- β_0, intercept
- $G(\underline{s}, t)$, a spatiotemporal GMRF
- ϵ, error

```
fit_lgcp(locs, sf, smesh, tmesh, parameters)
```

- `locs`, matrix of locations
- `sf`, an `sf` object of the spatial domain
- `smesh`, a `INLA::inla.mesh.2d` object
- `tmesh`, a `INLA::inla.mesh.1d` object
- `parameters`, vector of parameter starting values

A marked LGCP

A mark

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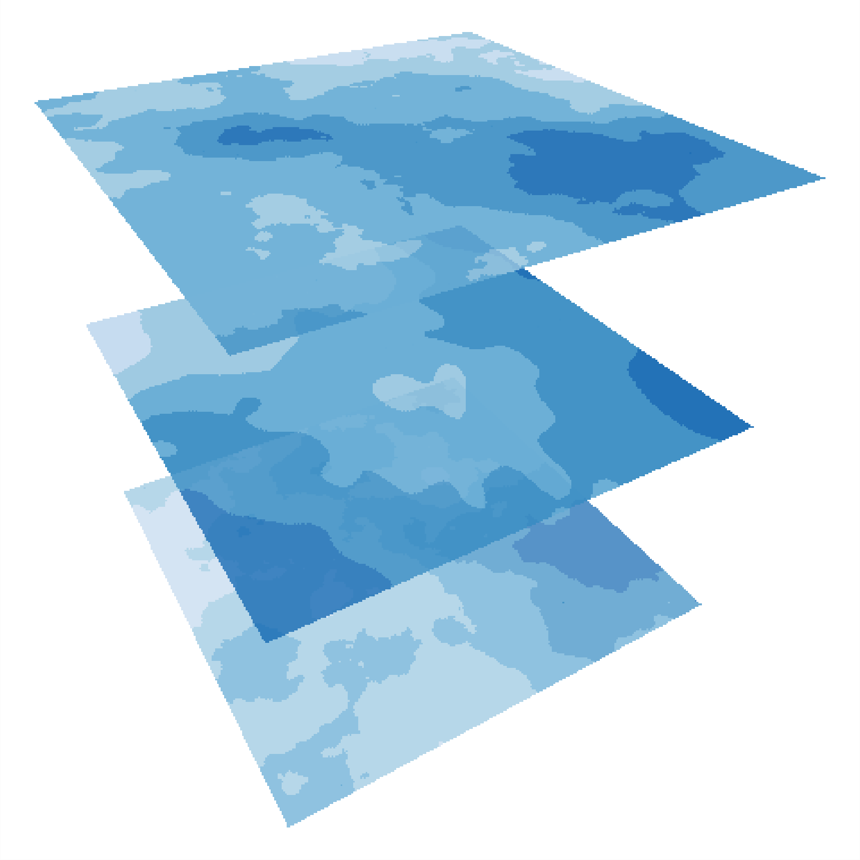


A marked LGCP

A mark

`fit_mlgcp`

- `marks`
- `metho`
- `fielc`
- `covar`

$$\Lambda_{pp}(s) = \exp(\beta_{pp} + G_{pp}(s) + \epsilon_{pp})$$

$$\Lambda_{m_j}(s) = f^{-1}(\beta_{m_j} + G_{m_j}(s) + \alpha_j\ G_{pp}(s) + \epsilon_{m_j})$$

- α_j ($j = 1, \ldots, n_{marks}$) are coefficient(s) linking the point process and the mark(s).
- $\Lambda_{m_j}(s)$ depends on the assumed distribution of the marks e.g.,

1. If $m_j \sim \text{Normal}(\mu_j(s), \sigma_j)$ then $M_j(s) = \mu_j(s)$ and $f^{-1} = I()$,
2. If $m_j \sim \text{Poisson}(\Lambda_j(s))$ then $M_j(s) = \Lambda_j(s)$ and $f^{-1} = \exp()$,
3. If $m_j \sim \text{Binomial}(n_j, p_j(s))$ then $M_j(s) = p_j(s)$ and $f^{-1} = \text{logit}()$, and
4. If $m_j \sim \text{Gamma}(\text{shape}_j(s), \text{scale}_j)$ then $M_j(s) = \text{shape}_j(s)$ and $f^{-1} = \log()$.

A marked LGCP

[redacted]

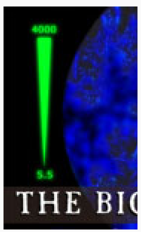
```
fit_mlgcp(locs, marks, sf, smesh, parameters, methods, fields, covariates)
```

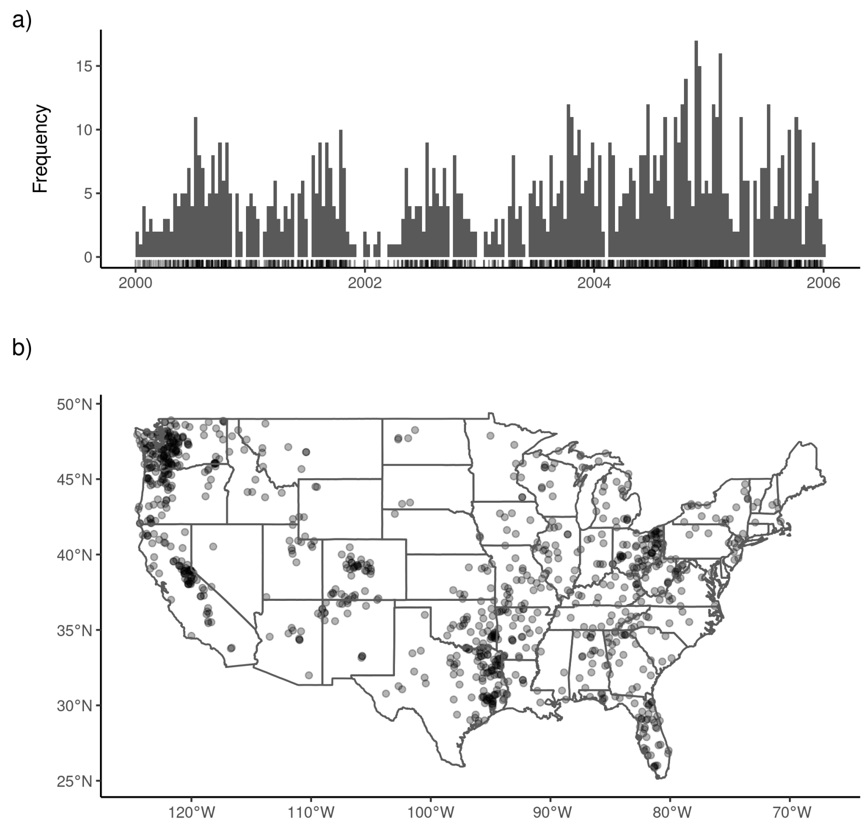
- `marks`, matrix of marks at each location in `locs`
- `methods`, integer(s) choice of mark distribution(s)
- `fields`, binary vector, should a mark specific random field be included
- `covariates`, a matrix of covariates can be set on either the marks, or LGCP, or both.

First to yell out

Sas

[redacted]



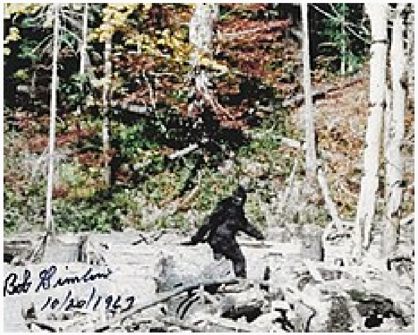


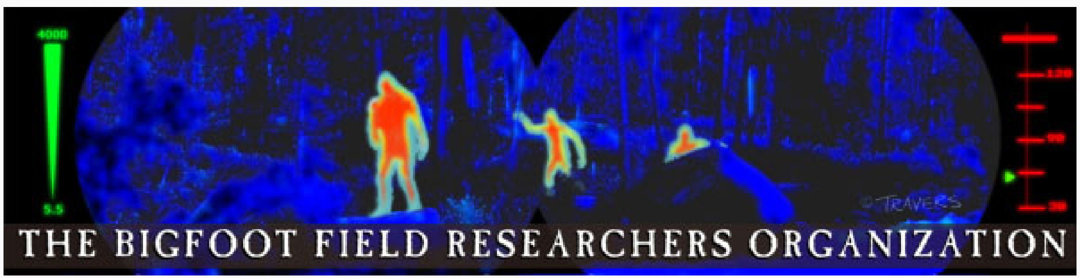
First to yell out the correct answer gets my permission to skip the queue @ lunch tomorrow

Sasquatch/Bigfoot

A mark

Setting \(m
sighting is





A marked LGCP example

Probab

Setting $m_i = 1$ if the Bigfoot sighting is classified as clear (Class A) and $m_i = 0$ if the sighting is not (class B).

The joint n

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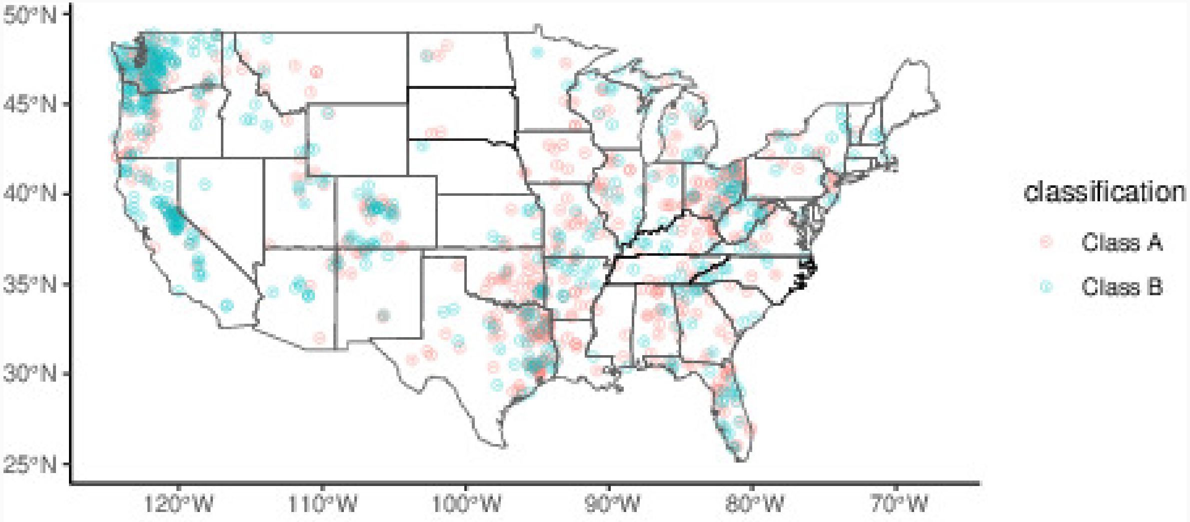


Probability of a first-hand sighing

A joint

Paramet

\(\beta_0\

\(\beta_1\

\(\beta_0^

\(\beta_1^

\(\alpha_m

The joint model we fit is

$$\Lambda(s) = \exp(\beta_0 + \beta_1 x_{elev}(s) + G(s) + \epsilon)$$

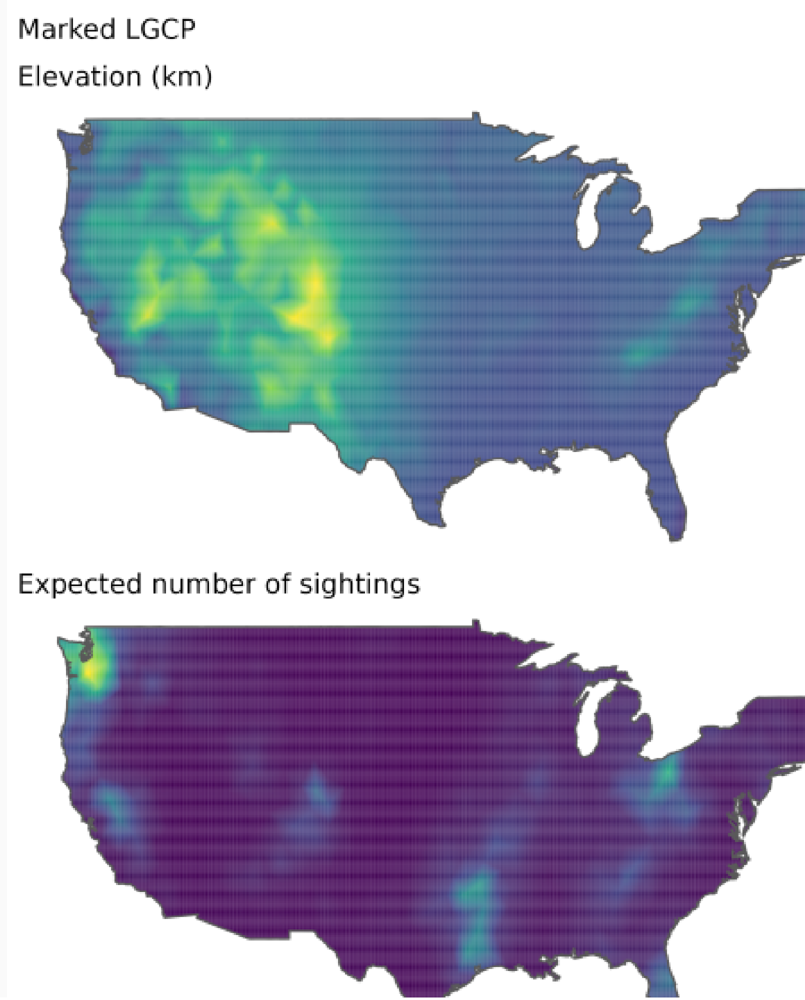
$$\text{logit}(p(s))^{-1} = \beta_{0^m} + \beta_1^m x_{elev}(s) + G_m(s) + \alpha_m\ G(s) + \epsilon_m$$

where $m(s) \sim \text{Bernoulli}(p(s))$ and the spatial intensity of all sightings is, as previously, $\Lambda(s)$. A spatial covariate, $x_{elev}(s)$ the elevation in kilometres

A joint model

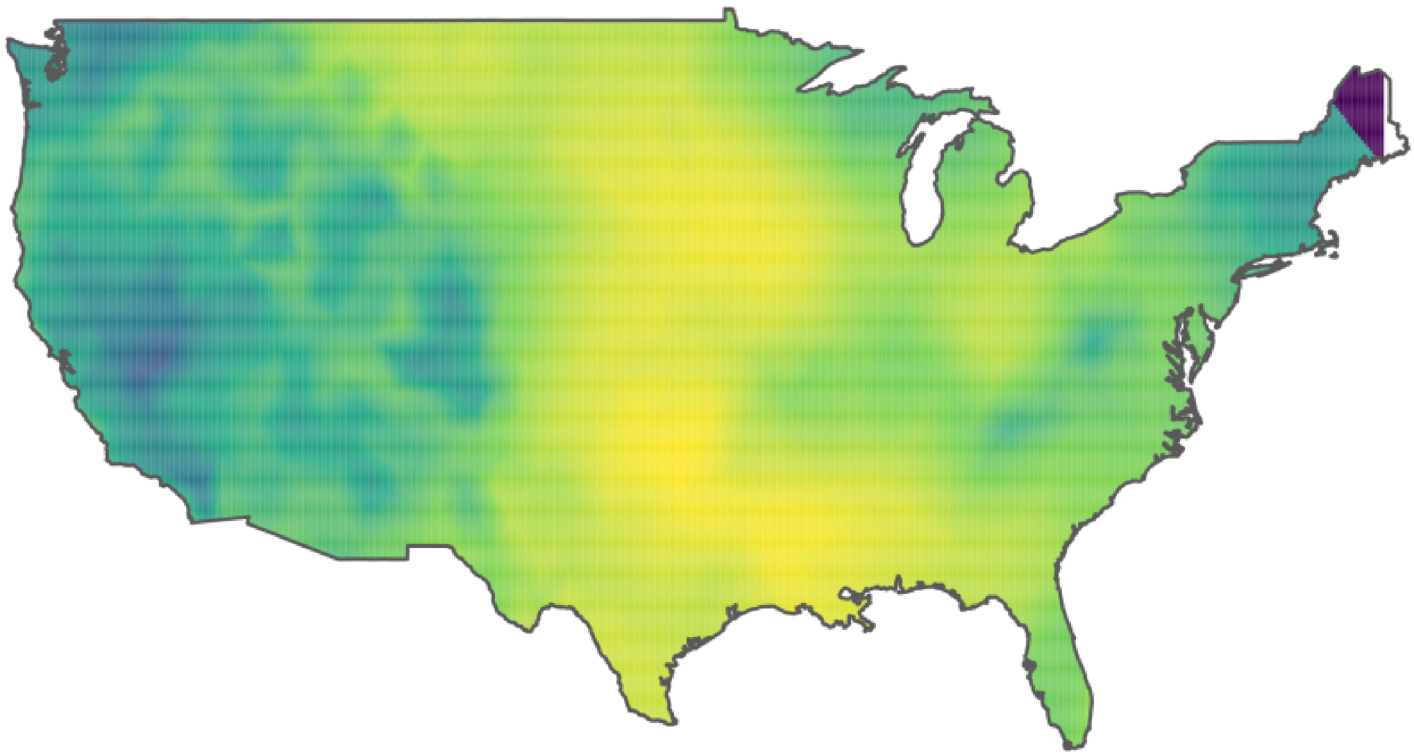
Probab

Parameter	est.	se
β_0	-0.823	0.358
β_1	0.114	0.251
β_0^m	0.227	0.284
β_1^m	-0.372	0.178
α_m	0.044	0.063



Probability of a first-hand sighing

But, is



But, is seeing Bigfoot contagious?

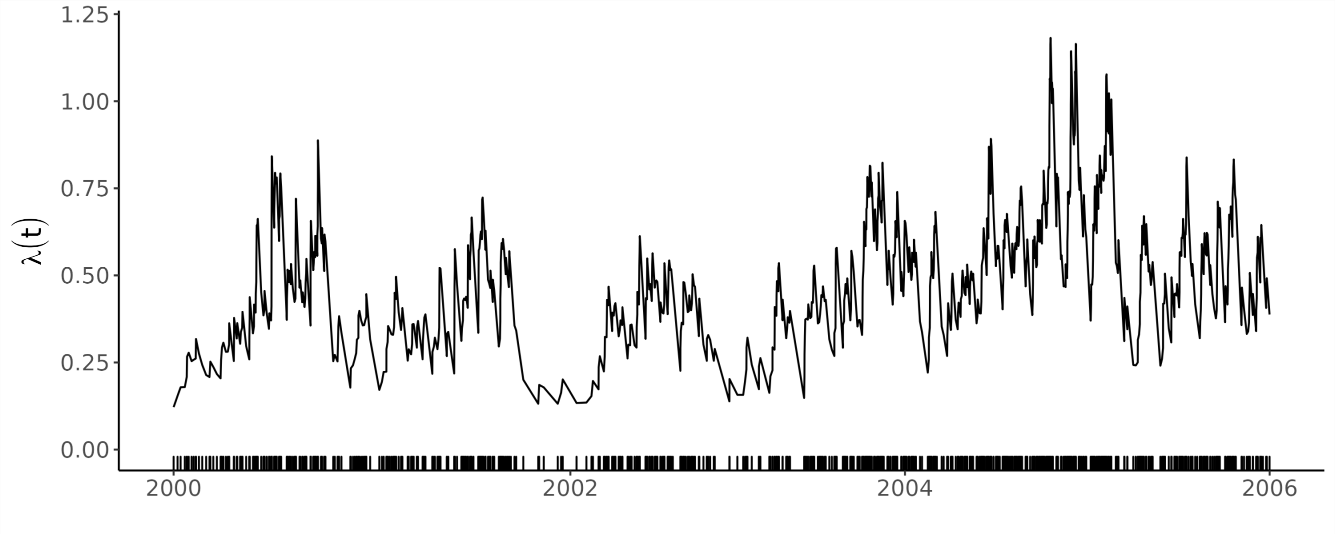
ETAS-t

$$\lambda

- \(\mu
- \(\alp
- \(\bet
- \(m(t)
- \(\Sig

`fit_hawke`

- `times`
- `param`
- `marks`



ETAS-type model

So, is s

$$\lambda(t; m(t)) = \mu + \alpha \Sigma_{i:\tau_i<t} m(\tau_i) \exp(-\beta * (t - \tau_i))$$

- μ, background rate
- α, increase in intensity after an event
- β, rate of decay
- $m(t)$, temporal mark
- $\Sigma_{i:\tau_i<t} \cdots$, historic dependence

```
fit_hawkes(times, parameters, marks)
```

- `times`, vector of times
- `parameters`, starting values of parameters
- `marks`, (optional) vector of marks

\$\$\lambda

- \(n = 9
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- Rate c

So, is seeing Bigfoot contagious?

Extens

$$\lambda(t) = \mu + \alpha\Sigma_{i:\tau_i<t}\exp(-\beta * (t - \tau_i)) + \epsilon$$

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- \(\mu

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- $n = 972$ sightings over $T = 2188$ days
- $\hat{\mu}T = 0.12 \times 2188 \sim 263$ baseline sightings
- Expected number of sightings triggered by any one sighting $\frac{\hat{\alpha}}{\hat{\beta}} = \frac{0.06}{0.09} = \frac{2}{3}$
- Expected number of descendants per sighting $\frac{\hat{\beta}}{\hat{\beta}-\hat{\alpha}} = \frac{0.09}{0.09-0.06} = 3$
- Rate of decay for the self-excitement $\frac{1}{\hat{\beta}} = \frac{1}{0.09} \sim 11$ days

Extension: An inhomogeneous Hawkes process

Extens

Condition

$$\lambda
\tau_i))}$$

- \(j, k \
- \(\alp

`fit_mhawk`

- `strea`

$$\lambda(t) = \mu(t) + \alpha \Sigma_{i:\tau_i < t} \exp(-\beta * (t - \tau_i))$$

- $\mu(t)$, varying background rate

```
fit_hawkes_cbf(times, parameters, background, background_integral,
background_param)
```

- `background`, user supplied $\mu(t)$
- `background_integral`, integral of `background`
- `background_param`, starting values of parameters for $\mu(t)$

Extension: A marked Hawkes process

Everyt

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Conditional intensity for the j^{th} ($j = 1, \ldots, N$)

$$\lambda(t)^{j*} = \mu_j + \Sigma_{k=1}^{N} \Sigma_{i:\tau_i<t} \alpha_{jk} e^{(-\beta_j * (t-\tau_i))}$$

- $j, k \in (1, \ldots, N)$
- α_{jk}, is the excitement caused by the k^{th} stream on the j^{th}

```
fit_mhawkes(times, stream, patameters)
```

- `stream`, character vector specifying the stream ID of each observation in `times`

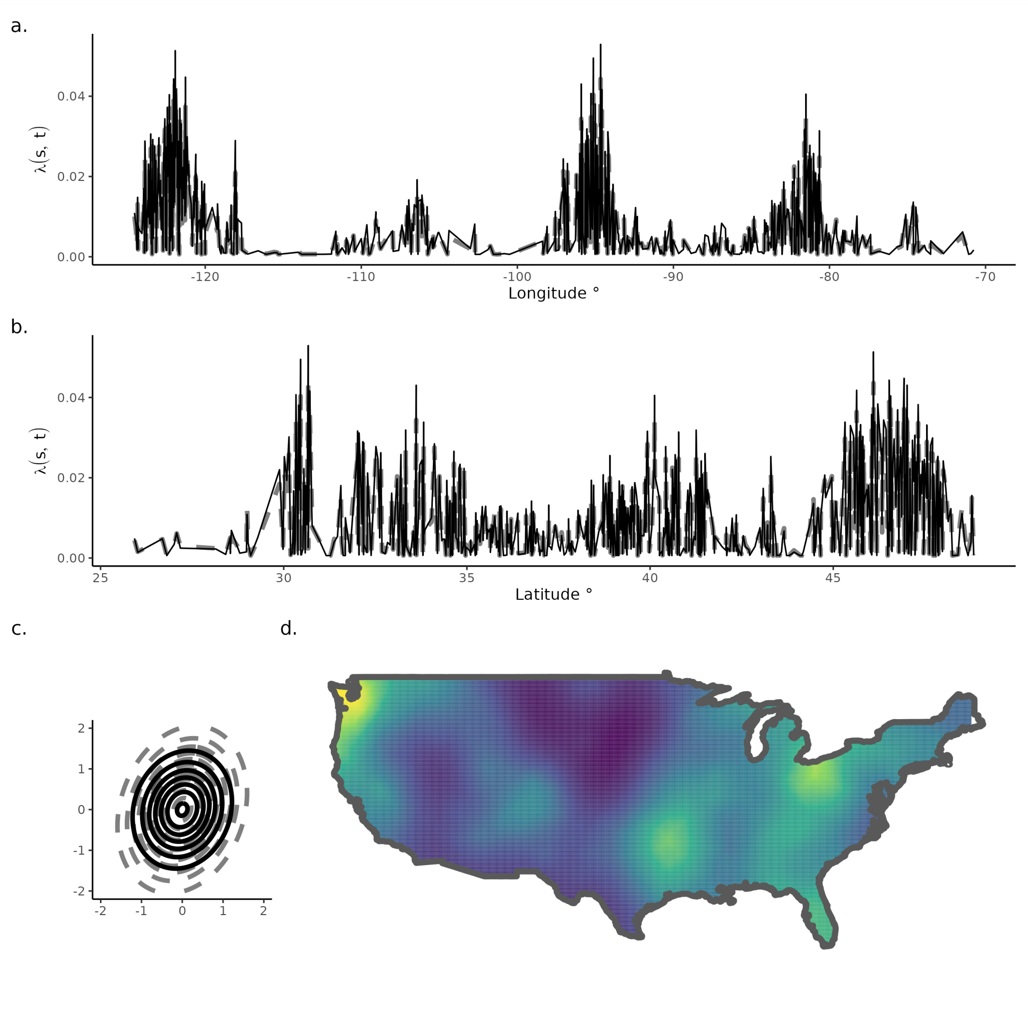
Everything in the pot: spatiotemporal self-excitment

$$\lambda(s,t) = \mu + G(s) + \alpha \Sigma_{i:\tau_i<t}(\exp(-\beta * (t-\tau_i))K_i(s-x_i, t-\tau_i)) + \epsilon$$

where μ is the background rate, β is the rate of temporal decay, α is the increase in intensity after an event, τ_i are the event times, and x_i are the event locations $G(s)$ is an (optional) Gaussian random field.

Spatial self-excitement kernel is given by $K_i(s-x_i, t-\tau_i) \sim \text{Normal}(0, Q^{-1})$, can either be time-independent where $Q^{-1} = [\begin{matrix} \sigma_x^2 & \rho\sigma_x\sigma_y \\ \rho\sigma_x\sigma_y & \sigma_y^2 \end{matrix}]$ or time-dependent where $Q^{-1} = [\begin{matrix} \sigma_x^2 & \rho\sigma_x\sigma_y \\ \rho\sigma_x\sigma_y & \sigma_y^2 \end{matrix}] \times (t_j - t_i)$ for $t_j > t_i$.

Spatiotemporal self-exciting process


a.
λ(s, t)
0.04
0.02
0.00
-120
-110
-100
-90
-80
-70
Longitude °
b.
λ(s, t)
0.04
0.02
0.00
25
30
35
40
45
Latitude °
c.
2
1
0
-1
-2
-2
-1
0
1
2
d.

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