# Categorising Ecological Community Count Data

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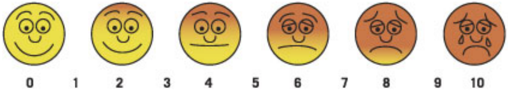
# Outline

1. Background.
   - Count data. Variance-mean ratio.
   - Approaches: Poisson, Negative Binomial.
   - Ordinal stereotype model.
2. Advantages of categorising count data into ordinal data.
3. Categorising count data: methodology.
4. Application
   - Spider data (Van der Aart & Smeenk-Enserink, 1974).
5. Summary.

# 1. Count and Ordinal data

- Count data:
  - Count the number times an event occurs, e.g. # particular species at a certain site.
  - Non-negative integers and zero being included or not (depending on whether it is ecologically important).
  - Counts may have **no upper bound**, or have a known maximum.
- Ordinal data:
  - Answers on ordinal variable describing inherent order.
  - The order in the response categories matters.
  - For example, Braun-Blanquet scale is very common in vegetation science or Likert scale in surveys.



# 1. Count data. Variance-mean ratio

- **Variance-mean ratio** $\left(\frac{\textbf{Var}}{\textbf{Mean}}, \textbf{VMR}\right)$. Stochastic scheme for classifying count data (Rogers, 1974, ch. 1)[1]:
  - VMR>1 (variance > mean) $\Rightarrow$ *clustered* point pattern.
  - VMR=1 (variance=mean) $\Rightarrow$ dispersion follows a *random* point pattern.
  - VMR<1 (variance < mean) $\Rightarrow$ *regular* point pattern.

a. Clustered b. Random c. Regular

1. Rogers, A. Statistical Analysis of Spatial Dispersion: The Quadrat Method. Monographs in Spatial and Environmental Systems Analysis. Pion, 1974.

# 1. Count data. Variance-mean ratio.

a. Clustered b. Random c. Regular

- *Clustered* point pattern ⇒ prob. object being in quadrat **linearly** related to # objects already there.
  e.g. shoal of sardines ⇒ **negative binomial distribution**.

- *Random* point pattern ⇒ prob. object being in quadrat **independent** of the # objects already there.
  e.g. plants with well-dispersed seeds ⇒ **Poisson distribution**

- *Regular* point pattern ⇒ prob. object being in quadrat **decreases linearly** with the # objects there.
  e.g. gannet nests in a colony ⇒ **binomial distribution**.

# 1. Count data. Variance-mean ratio

a. Clustered b. Random c. Regular

- Variance-mean relationship is a critical property of count data.
- Trends in location (mean abundance) may be confounded with changes in dispersion (Warton *et al.*, 2012)[2]
- One **alternative** to deal with VMR problem $\Rightarrow$ turn count data into ordinal.

2. Warton, D. I., Wright, S. T., and Wang, Y. Distance-based multivariate analyses confound location and dispersion effects. Methods in Ecology and Evolution, 3(1):89-101, 2012.

# 2. Advantages of categorising count data

- **Possible drawbacks** what could arise from using count data.
  1. Highly sensitive to **outliers** $\Rightarrow$ negative binomial.
  2. Structurally **exclude zero counts** (e.g. hospital length of stay (in days)) $\Rightarrow$ zero-truncated models.
  3. **Excess of zero counts** $\Rightarrow$ hurdle models, zero-inflated models.
  4. One data set with **different levels of VMR** $\Rightarrow$ apply different count data models.

- **Advantages of categorising** count data into ordinal categories:
  1. Less sensitive to <u>outliers</u>.
  2. No affected by the <u>omission of zeros</u> in the data.
  3. <u>Excess of zero counts</u> $\Rightarrow$ Cumulative link random effects models $\Rightarrow$ more parsimonious (Agresti, 2010).
  4. Use of the same approach for <u>different levels of VMR</u>.

# 1. Approaches

- Data represented as a matrix $Y$ with dimensions $n \times p$ ($n$ could be sites, $p$ could be spp.)

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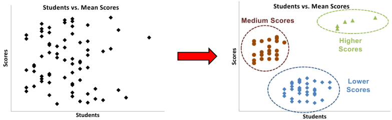
- Single-mode **clustering** and biclustering $\Rightarrow$ **Finite mixture models**.

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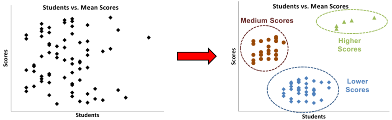


- Missing information: row/col membership $\Rightarrow$ EM algor., RJMCMC
- Count data sets:
  - Assumption of Poisson distribution (Pledger and Arnold, 2014)[3]
  - Negative binomial distribution when overdispersion.

- Ordinal data sets (after categorising):
  - Assumption of ordinal stereotype model (Fernández *et al.*, 2014)[4]

3. Pledger, S. and Arnold, R. Multivariate methods using mixtures: Correspondence analysis, scaling and pattern-detection. Computational Stat. and Data Analysis, 2014.

4. Fernández, D., Arnold, R., and Pledger, S. Mixture-based clustering for the ordered stereotype model. Computational Stat. and Data Analysis, 2014.

# 3. Methodology. How Many Ordinal Categories?

Several ways of categorising:

- Simplest case: Using count data as ordinal categories (e.g. $(0, 1, 2, 3) \Rightarrow \{0, 1, 2, 3\}$).

- Large counts. Use top-coded data (e.g. $\{0, 1, \mathbf{2+}\}$). $\{0, 1+\}$: presence-absence, extreme case.

- Equally spaced cut points (e.g. $0 - 4, 5 - 9, \ldots$ or $0, 1 - 9, 10 - 99, \ldots$ with logarithmic scale).

- Replace count data by their ranks and cutting them into groups based on **percentiles**.
  - Percentiles are not strongly influenced by extreme values
  - Can be calculated even if the counts are skewed.

# 3. Methodology. Categorising Based on Percentiles

Given count data $\{y_{ij}\}$ ($i = 1, \ldots, n$ and $j = 1, \ldots, p$).

1. **Rescale** each observation $y_{ij}$, so $y_{ij}^{\mathrm{st}} \in [0, 1]$.
2. **Divide** vector $\{y_{ij}^{\mathrm{st}}\}$ into $\ell + 1$ quantiles: $Q^{(0)}, \ldots, Q^{(\ell)}$.
3. **Recode** each observation $y_{ij}^{\mathrm{st}}$ as:

$$y'_{ij} = \begin{cases} 0 & \text{if } y_{ij}^{\mathrm{st}} \in [Q^{(0)}, Q^{(1)}], \\ k & \text{if } y_{ij}^{\mathrm{st}} \in (Q^{(k)}, Q^{(k+1)}], \end{cases}$$

   where $(Q^{(k-1)}, Q^{(k)}]$ is the interval of values from vector $y_{ij}^{\mathrm{st}}$ between the $(k-1)^{\mathrm{th}}$ and $k^{\mathrm{th}}$ quantiles, for $k = 1, \ldots, \ell$.
   Each interval contains $\frac{100}{\ell}\%$ of the non-zero data.
4. **Fit** our ordinal mixture methodology to $\mathbf{Y'}$.

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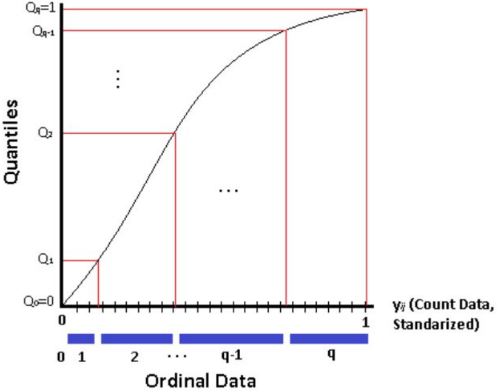


# 4. Application. Spider Dataset


*Pardosa monticola* - pin-stripe wolf spider, and it inhabits sand dunes in the Netherlands.

# 4. Application. Spider Dataset

- "Spider" abundance data (Van der Aart & Smeenk, 1974).
- 12 spider species , 28 sites .
- Original data: Count data (species abundance at site).
- Ordinal data: 4 categories.

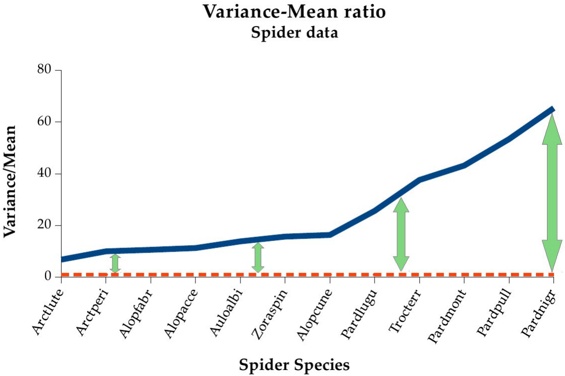
$$y_{ij} = \begin{cases} (0)\ \textbf{None} & \text{No data recorded} \\ (1)\ \textbf{Low} & \text{Species coverage is below 25\%} \\ (2)\ \textbf{Medium} & \text{Species coverage is between 25\% -- 65\%} \\ (3)\ \textbf{High} & \text{Species coverage is higher than 65\%} \end{cases}$$

Table : Frequencies of spider abundance by site, in 4-level ordinal scale.

| **Ordinal scale** | **0** | **1** | **2** | **3** | **Total** |
|---|---|---|---|---|---|
| Spider abundance | No data recorded | Low | Medium | High | Total |
| Frequency ($y_{ij}$) | 154 | 66 | 56 | 60 | 336 |

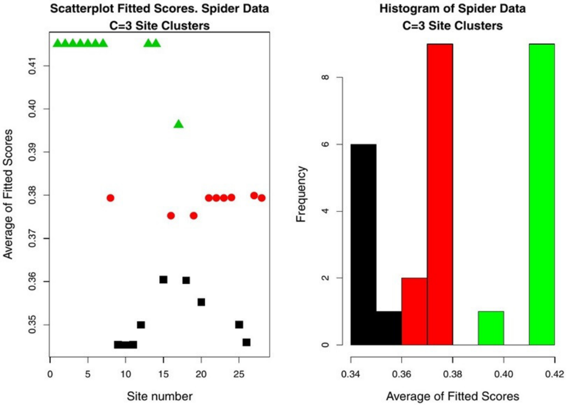
# 4. Application. Spider Dataset

Blue line: Variance-mean ratio (sorted ascending) for the spider data set.
Orange dashed line: indicates no overdispersion.
Green arrows: **Overdispersion** (variance$>$mean) is observed in all the species.



# 4. Application. Spider Dataset

Scatter plot and histogram of the $R = 3$ fitted sites clusters $\{\overline{\phi}_{(i.)}\}$ from the row clustering version of the stereotype model $(\mu_k + \phi_k(\alpha_r + \beta_j))$.



# Cluster Results
# Count data vs. Ordinal data

# 3. Result Comparison: Count data vs. Ordinal data

Table : **Spider data set**: Site clustering results for Poisson, NB and ordered stereotype model.

| Groups | **Clustering (highest probability)** | | |
|---|---|---|---|
| | **Poisson** | **NB** | **Stereotype** |
| R1 | {**1-7**,9-12, **13, 14**, 25} | {**1-7,13,14**} | {**1-7,13,14**} |
| R2 | {22-24,26-28} | {9-12,22-28} | {8,21-24,27,28} |
| R3 | {8,15-21} | {8,15-21} | {9-12,15-20,25-26} |

# 3. Results Column Clustering. Comparison

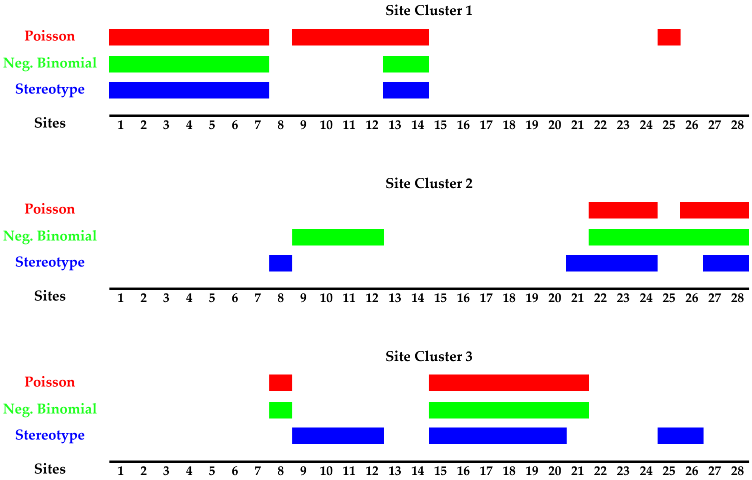



Figure : **Spider data C=3**: Poisson, Neg. Bin. and Ordinal Stereotype

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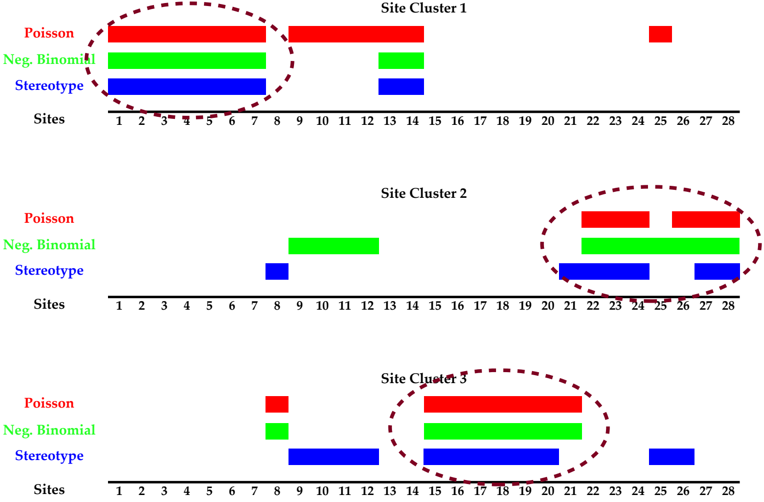


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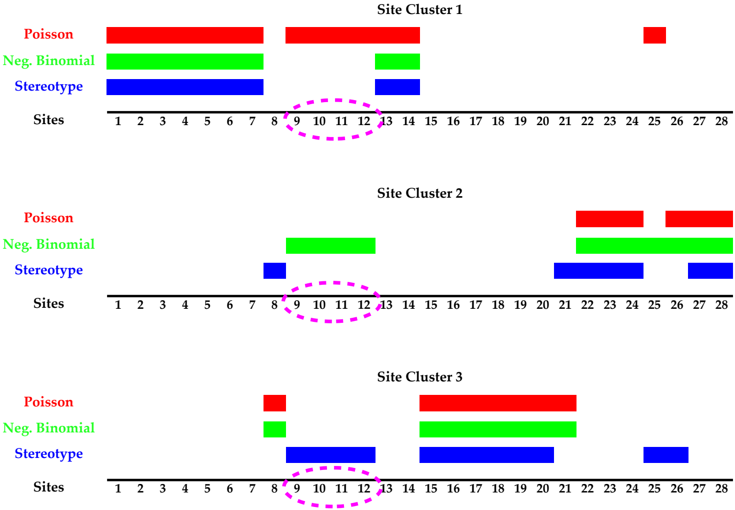


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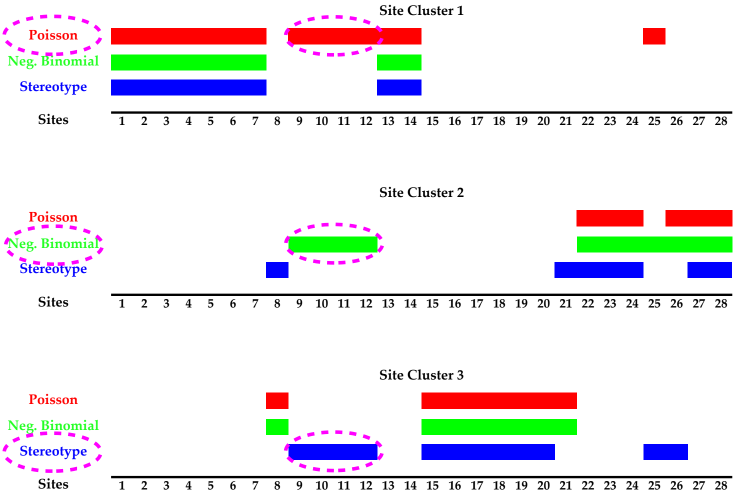


Figure : **Spider data C=3**: Poisson, Neg. Bin. and Ordinal Stereotype

# 3. Results Column Clustering. Comparison

- Clustering measures:
  - Variation of information (VI, Meila (2005)).
  - Normalized information distance (NID, Kraskov *et al.* (2005)),
  - Adjusted Rand index (ARI, Hubert *et al.* (1985))
- Range between (0,1).
- Large values indicate similarity of clustering.

Table : **Spider data set**: Clustering results for Poisson, NB, and stereotype model. Stereotype is closer to NB than Poisson.

| Clustering Comparison | ARI | 1-NVI | 1-NID |
|---|---|---|---|
| Poisson vs. Stereotype | 0.334 | 0.304 | 0.457 |
| **NB vs. Stereotype** | **0.465** | **0.423** | **0.590** |

# 5. Summary. Conclusions

- Features of categorising count data into ordinal data were shown.
- In our view, advantages:
  - We **do not have** to decide among different parametric models for the data. (i.e. it enables the inclusion of all of the different levels of dispersion in one methodology.)
  - Replacing high/low counts with "high/low" ordinal categories makes the actual **counts less influential** in the model fitting.
  - **Saving in cost** of sampling time in collecting only ordinal data (sample more sites).
- Future research directions:
  - Numerical experiment: Investigate the differences between recoded and original count data.
  - Developing a measure to quantify the loss of information.

# Acknowledgments and References

- Shirley Pledger and Richard Arnold.
- Funding: Victoria University of Wellington.

1. Rogers, A. Statistical Analysis of Spatial Dispersion: The Quadrat Method. Monographs in Spatial and Environmental Systems Analysis. Pion, 1974.
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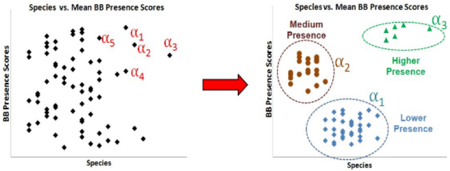
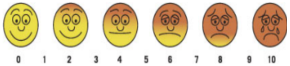
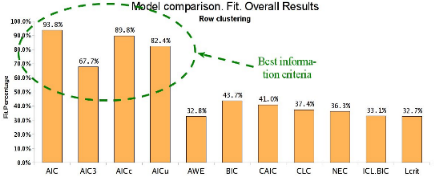
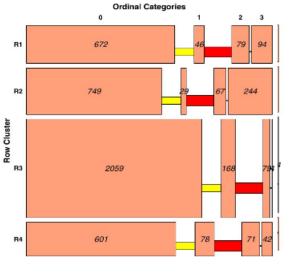
# Thanks for listening!!!



# References

## Questions?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# Extra Slides

# 1. Approaches. Ordinal stereotype model

For example, **Row clustered ordinal stereotype model**:

$$\log\left(\frac{P\left[y_{ij}=k \mid i \in r\right]}{P\left[y_{ij}=1 \mid i \in r\right]}\right)=\mu_k+\phi_k(\alpha_r+\beta_j)$$

$$i=1,\dots,n \quad j=1,\dots,p \quad k=1,\dots,q \quad r=1,\dots,R<n$$

- $\mu_k$: cut points (nuisance parameters).
- $\alpha_r$: effect of the row cluster $r$.
- $\beta_j$: effect of the columns.
- $\phi_k$: "score" for the response category k.

- Including an increasing order constraint:

$$0=\phi_1 \leq \phi_2 \leq \dots \leq \phi_q = 1 \ ,$$

captures the ordinal nature of the outcomes.