Fitting linear mixed models under misspecification

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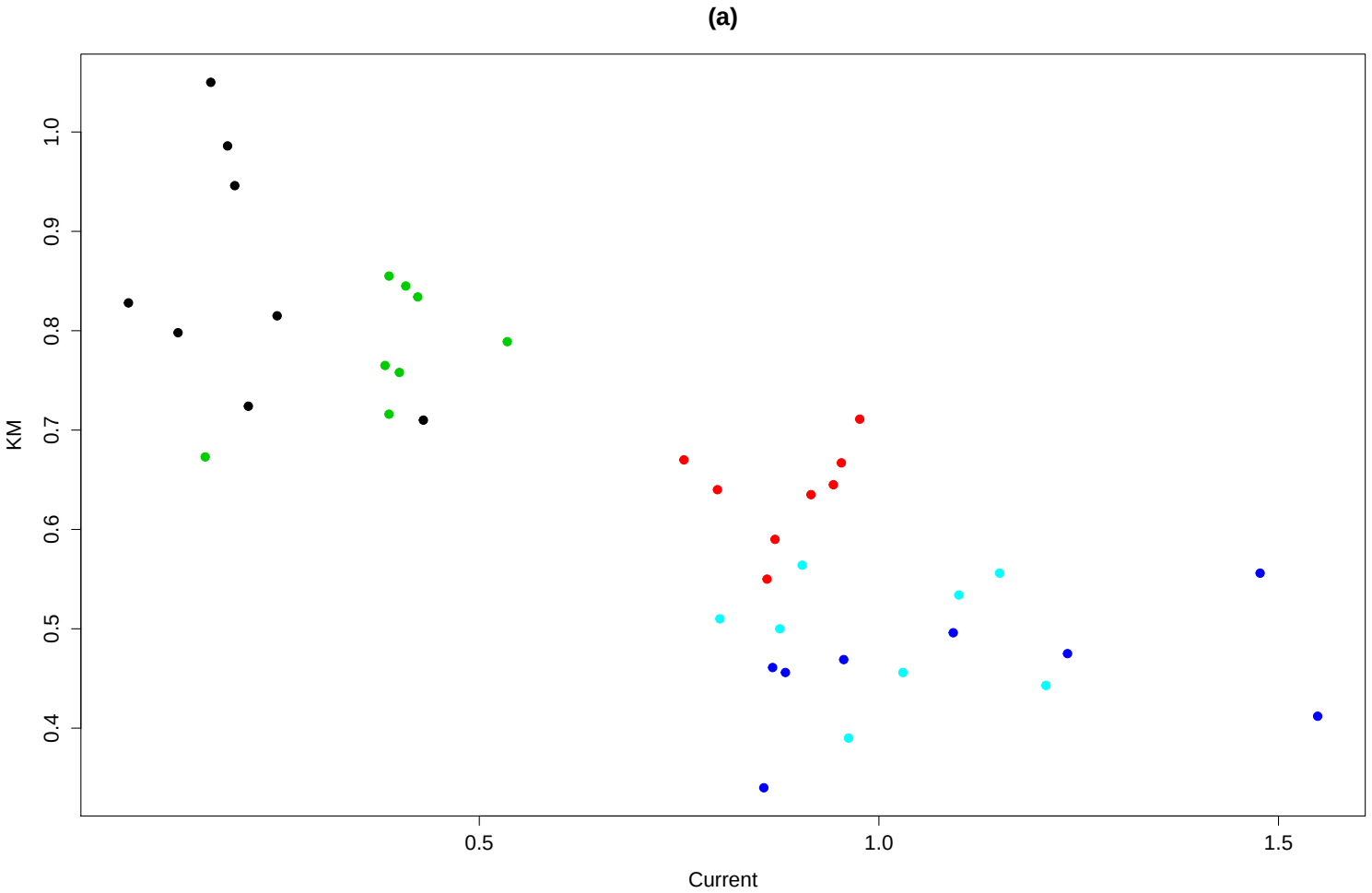
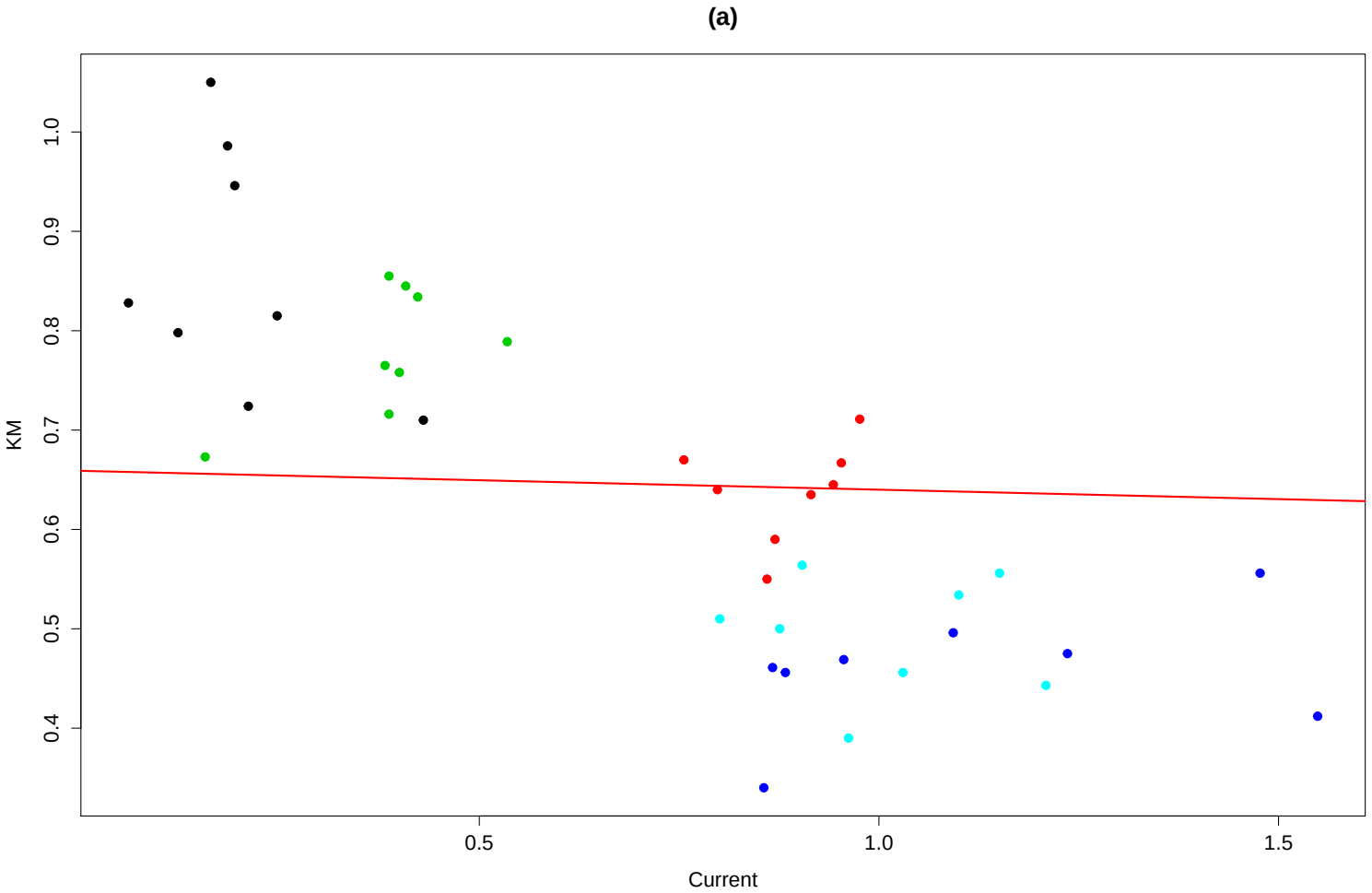
How much believe in LMM ?

- Linear mixed model (LMM) is one of the most popular statistical methods and being used without a doubt
- What if LMM does not give right estimates ?

Examples



Examples



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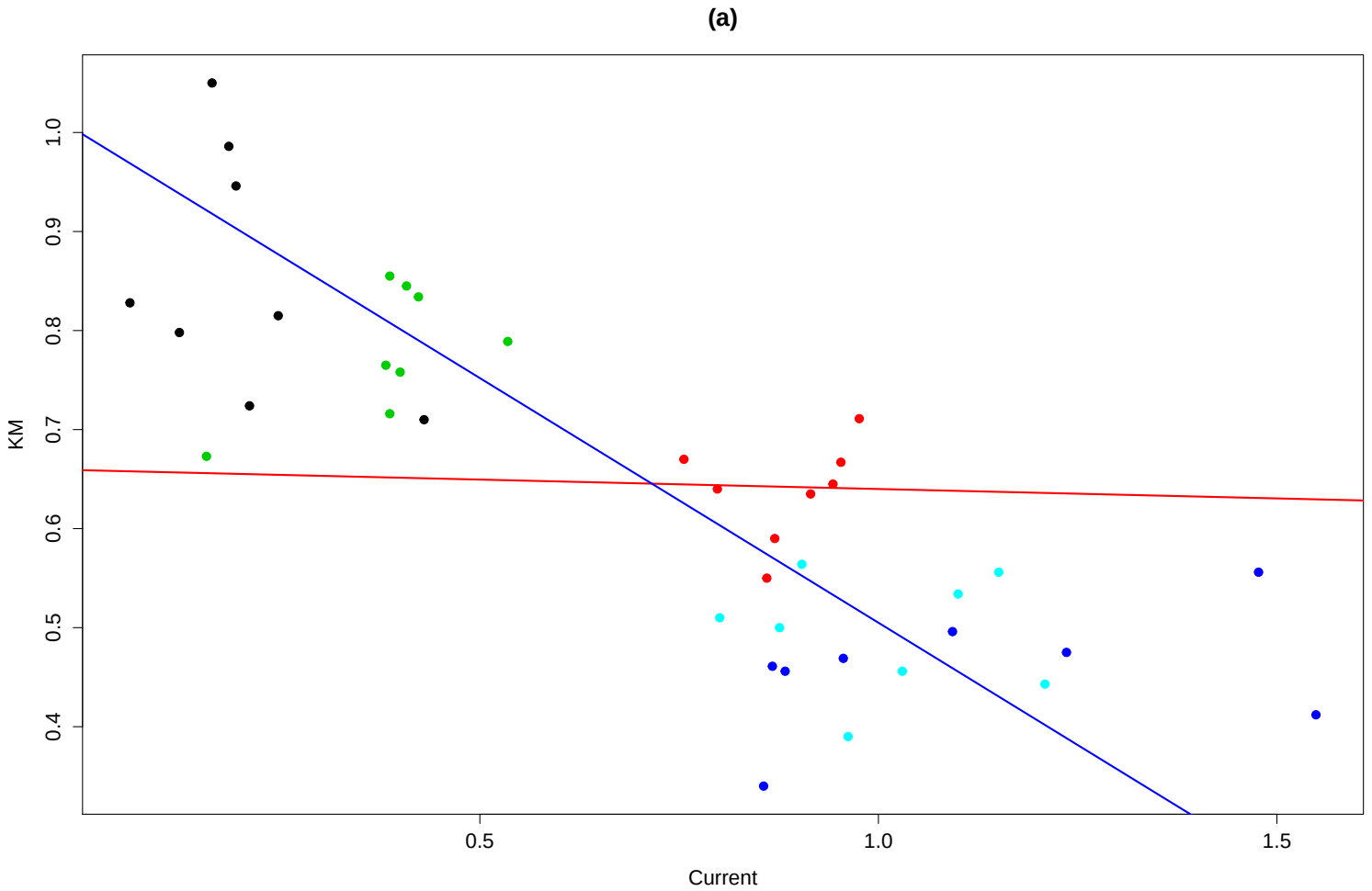
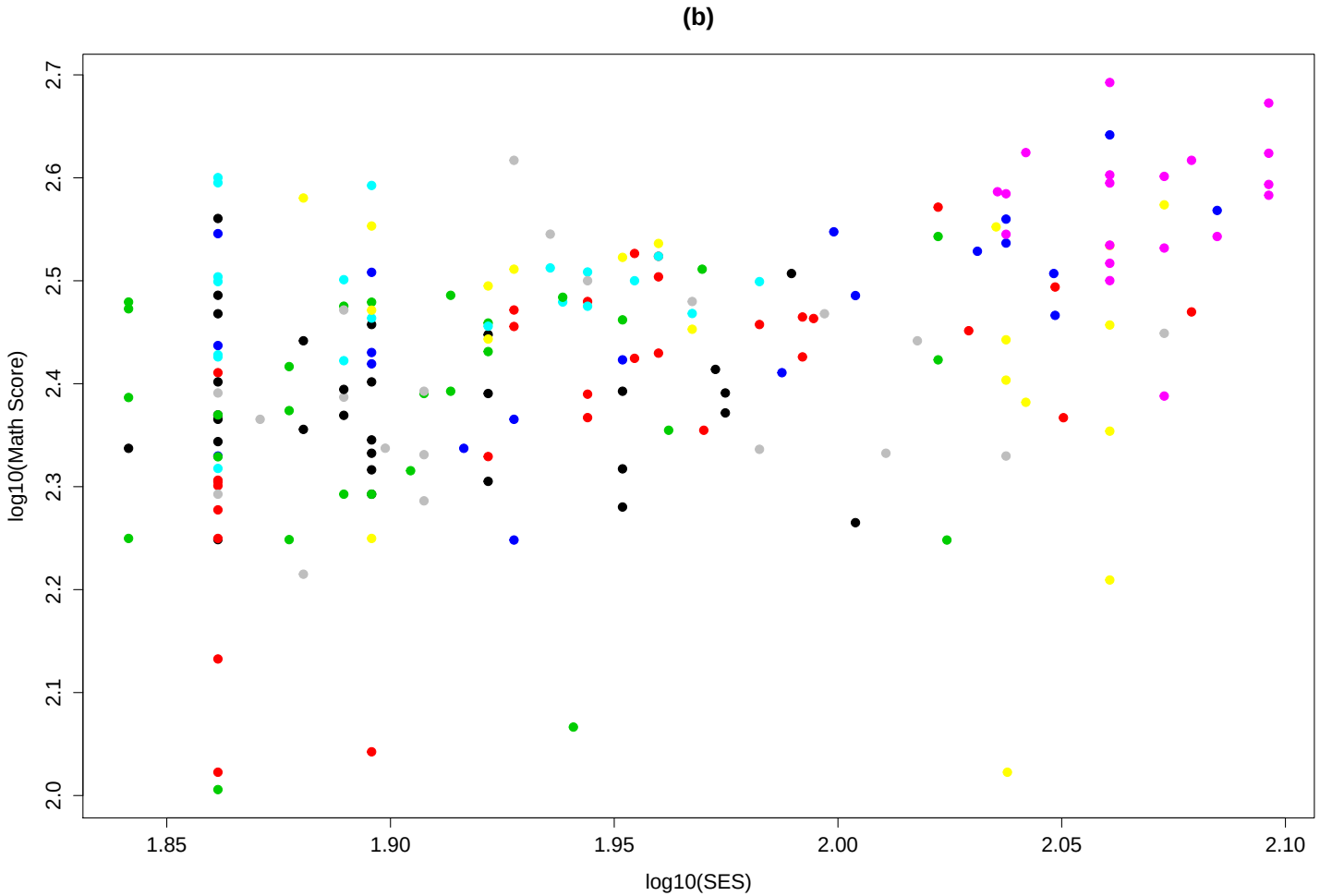


Examples (a)

- Enzyme kinetics data
- $n = 8$ measurements on four mutant mice and one wild type mouse ($m = 5$) of kinetics (Y) and the current (X)
- REML based on lme4 in R and REML on our method
- Between cluster variance of the current (X) is $\hat{\tau}_x = 0.397^2$

Examples


(b)
log10(Math Score)
2.0
2.1
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1.90
1.95
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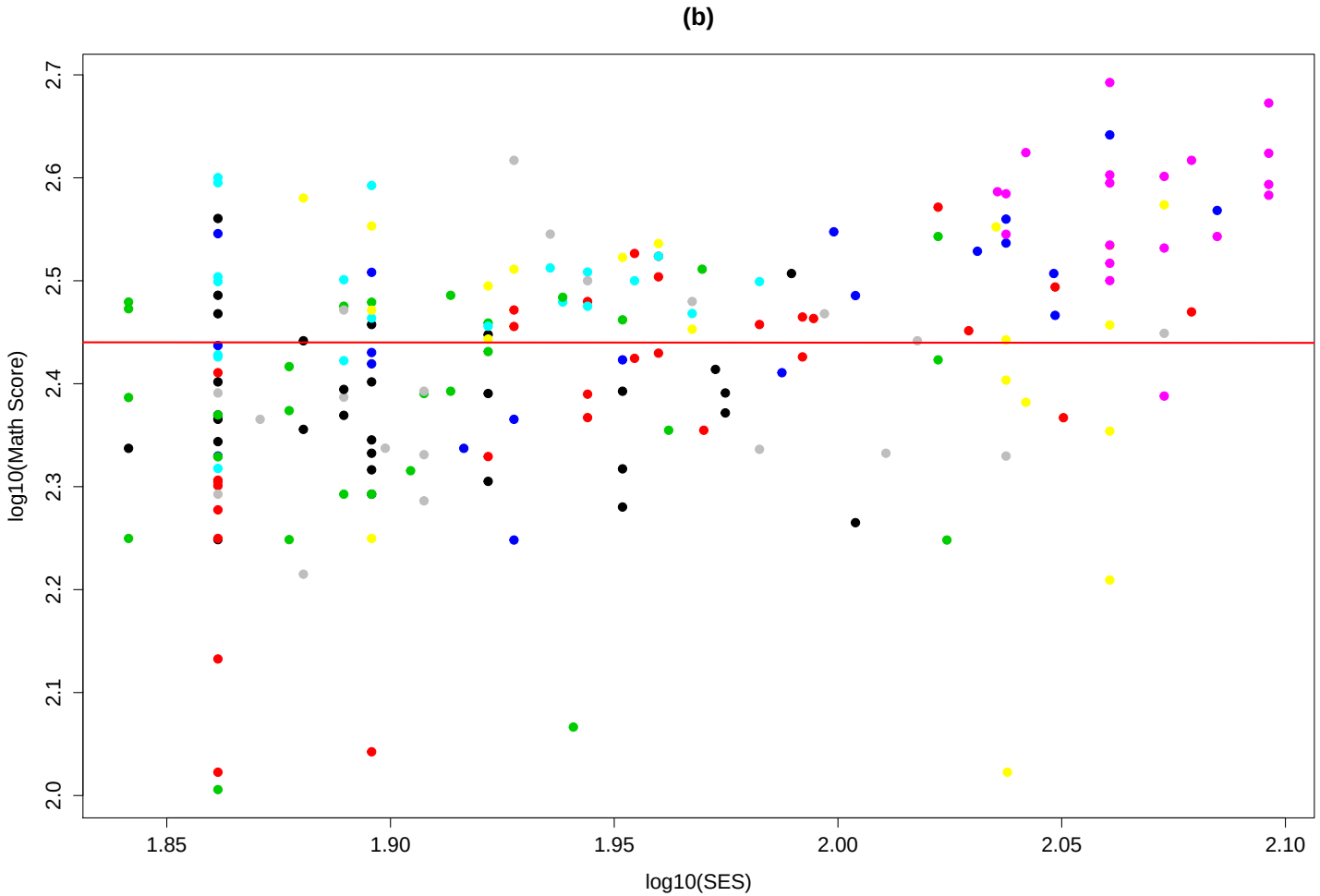
2.05
2.10
log10(SES)

Examples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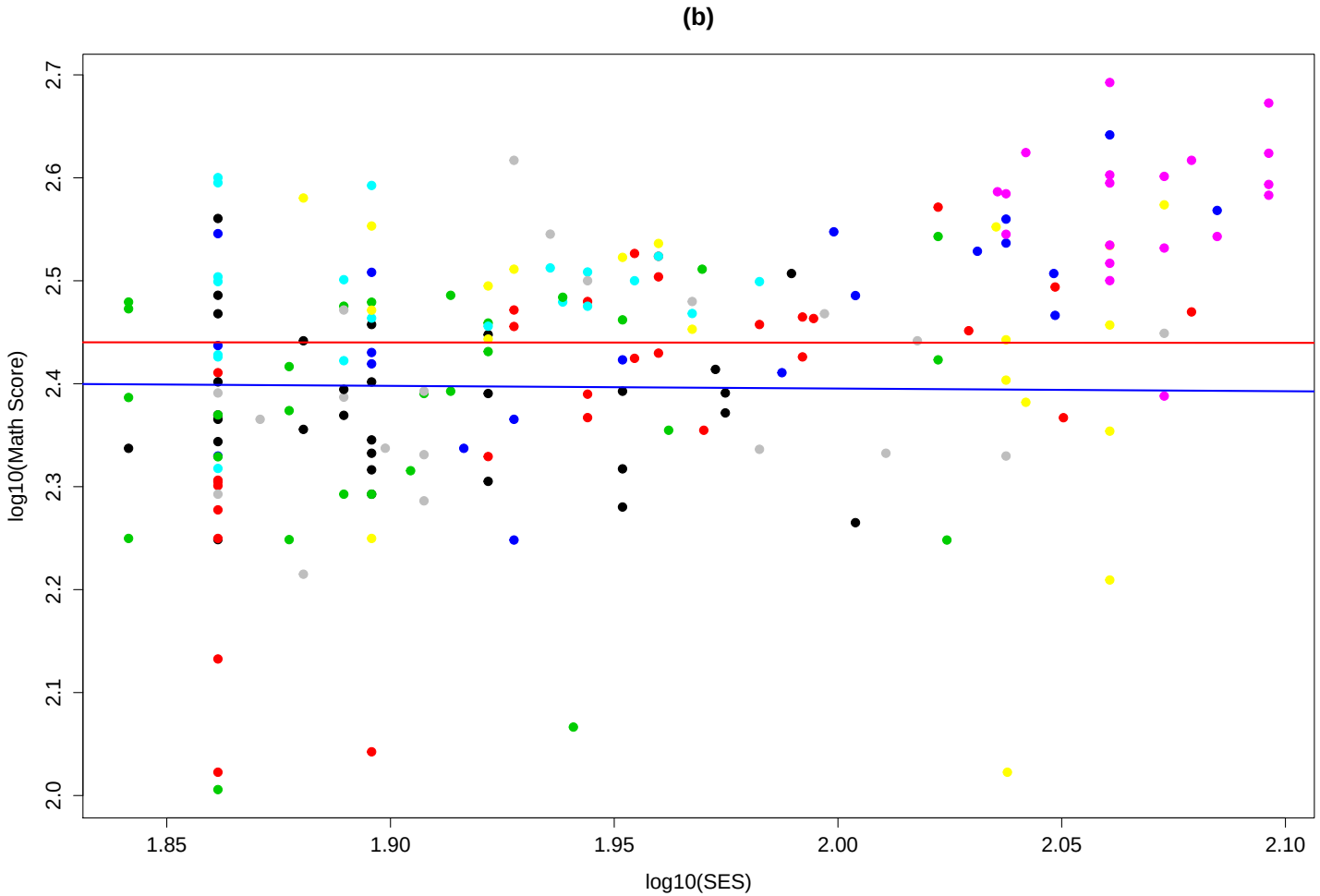
Examples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2.05
2.10
log10(SES)

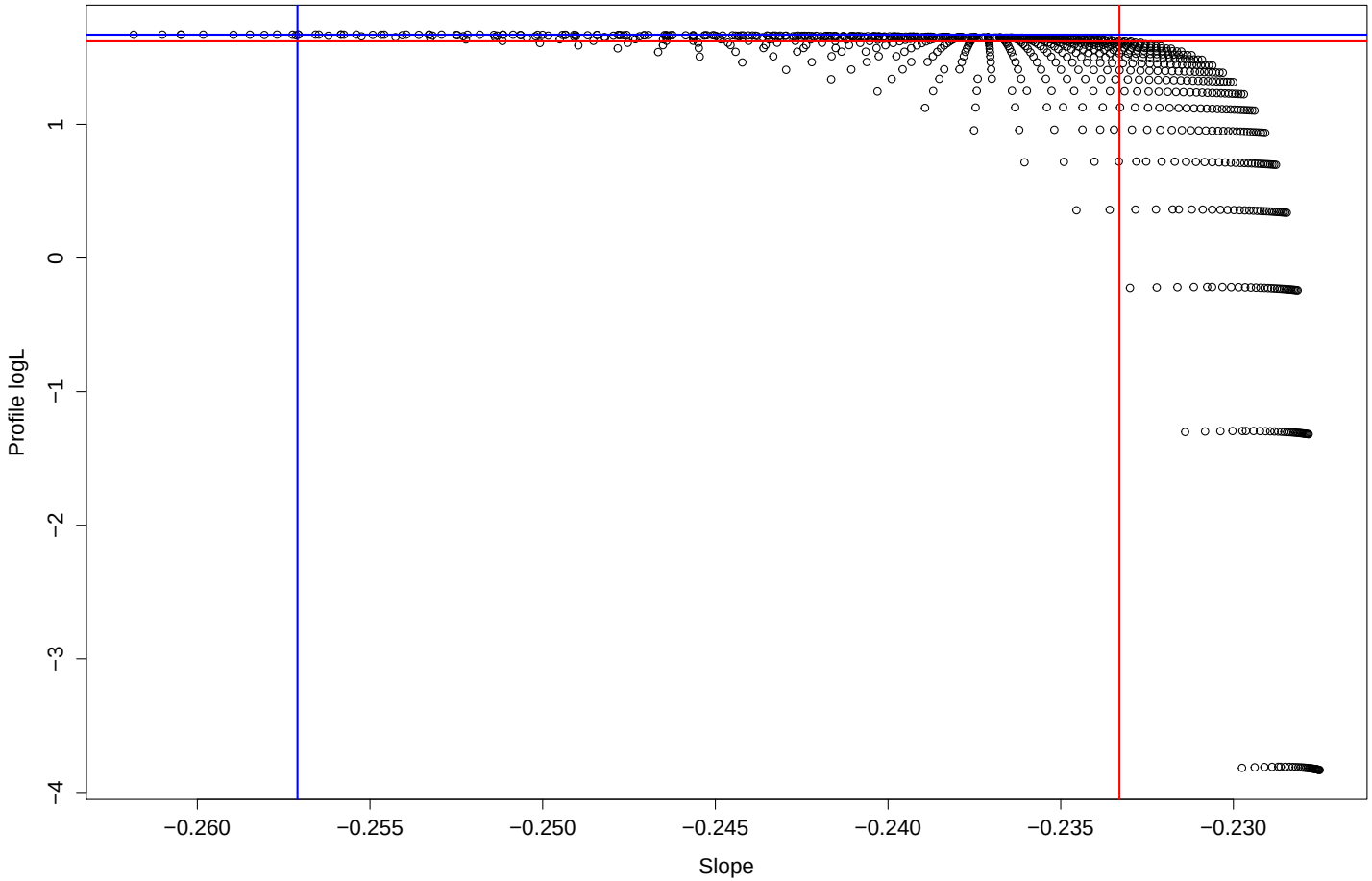
Examples (b)

- Sixth Grade data
- $n = 10$ measurements on $m = 19$ different schools of mathematics score (Y) and the socio-economic status (X)
- REML based on lme4 in R and REML on our method
- Between cluster variance of the socio-economic status (X) is $\hat{\tau}_x = 14.645^2$

Examples cont.

- Why ? $\rightarrow$ fit misspecified model: unintentionally, because the true model is unknown and intentionally in either informal model exploration or formal model selection

Profile log-likelihood (a)



Linear mixed model

- Classic LMM

$$Y_{ij} = \beta_0 + \beta_c X_{ij} + \delta_i + \epsilon_{ij},$$

where $(\beta_0,\ \beta_c)$ the unknown regression parameters and $(\delta_i,\ \epsilon_{ij})$ an unobserved random intercept and random error.

LMM cont.

- Contextual LMM ($\tilde{X}_{ij} = X_{ij} - \bar{X}_i$)

$$Y_{ij} = \beta_0^* + \beta_w \tilde{X}_{ij} + \beta_b \bar{X}_i + \delta_i^* + \epsilon_{ij}^*,$$

where (β_w, β_b) the within- and between-group regression parameters and (δ_i^*, ϵ_{ij}^*) an unobserved random intercept and random error.

Misspecification

- When $\beta_b - \beta_w \neq 0$, LMM is missepecified due to omitting the group level confounder $\bar{x}_i$ that is, clustering in x_i is ignored.
- Affects the estimates of the variance components not just, the estimates of the regression parameters

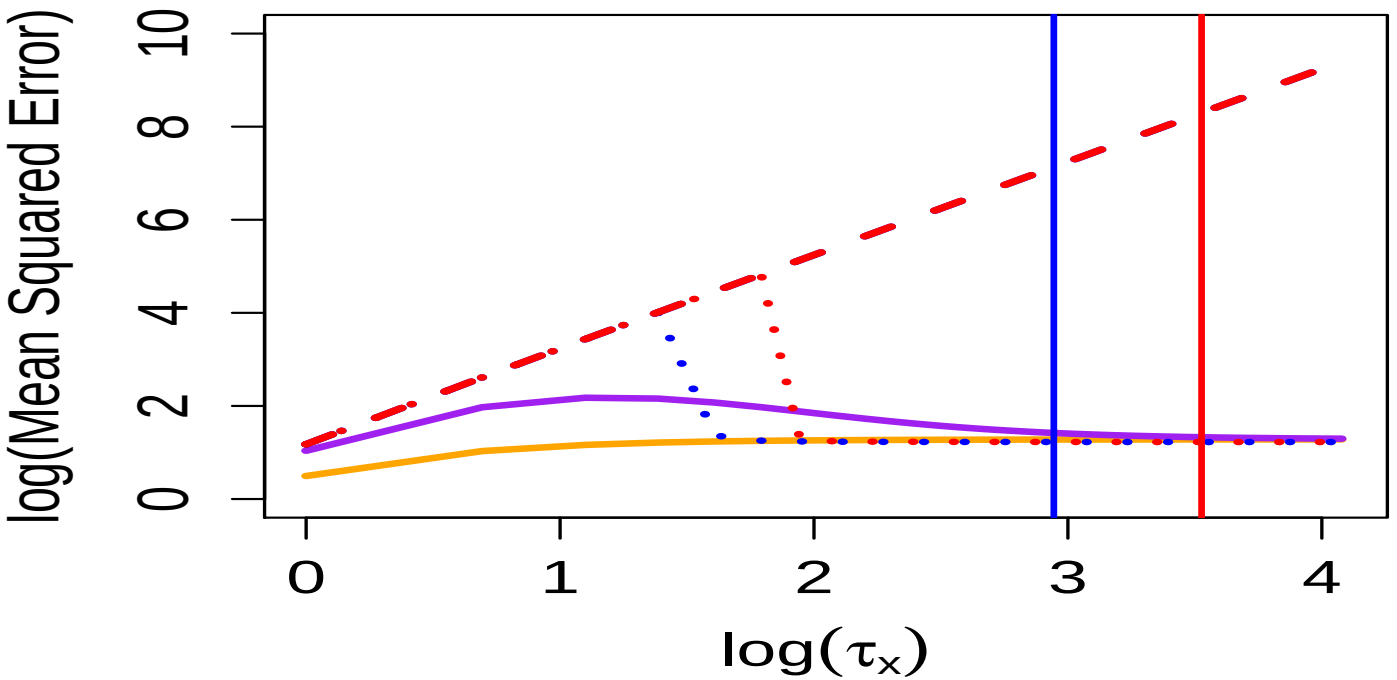
Result

- Fitting LMM when CLMM is true model can lead to very misleading assessments of the association between Y and X
- Change LS and WLS estimators of the regression parameters smoothly as changing between cluster variance of X, τ_X
- but jump ML and REML estimators of the regression and variance components.

Result cont.

- The reason for the jumps in ML and REML estimators: as $\tau_x \uparrow$, the ML and REML criterion develop two distinct local maxima and which of these is the global maximum changes at the jump point.

Mean squared errors



Thank you !